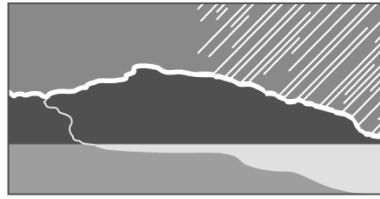


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CATCHMENT HYDROLOGY

WORKING DOCUMENT

GENERATION OF SPATIALLY AVERAGED DAILY RAINFALLS FOR THE YARRA REGION

**L. Siriwardena
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ABSTRACT

This document describes the data preparation and the generation of areal average rainfall for Yarra catchment. Two daily rainfall generation models, the Transition Probability Matrix (TPM) model and a modified Wang-Nathan Model (WNM), were used to derive spatially averaged daily rainfall sequences for a region encompassing Yarra catchment in Victoria, one of the focus catchments in the CRC for Catchment Hydrology. The performance of the two data generation models was evaluated with respect to their ability to preserve various important rainfall characteristics at daily, monthly and annual time scales.

It was shown that the modified WNM is superior to TPM model with respect to preserving the mean, standard deviation and skew of the historic rainfall at daily, monthly and annual time scales. Dry spell and wet spell characteristics are modeled satisfactorily by both models with slightly improved performance with the modified WNM. However, both models are incapable of preserving the monthly variation of serial correlation.

Overall, the modified Wang-Nathan Model is shown to be the better daily generation model based on testing carried out for Yarra region.

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1. INTRODUCTION

The work reported here covers part of the Project 5.2 of CRC for Catchment Hydrology – “National Data Bank of Stochastic Climate and Streamflow Models”. The project aims to develop a robust set of stochastic models for generation of rainfall, streamflow and other climatic data. This report is focused on the stochastic generation of daily rainfall data.

There has been a steadily increasing interest in the stochastic generation of long sequences of daily rainfalls for accurate assessment of extreme droughts and floods. Srikanthan and McMahon (1985) reviewed daily rainfall generating models, and they describe the various approaches which have been reported in the literature. Chapman (1994) tested 5 daily rainfall generation models using different methods of evaluating the model parameters, and reported that the Srikanthan-McMahon Transition Probability Matrix (TPM) model performed particularly well when calibrated with long rainfall records. Boughton (1999) tested an improved TPM model for daily rainfall generation and concluded that the model performs satisfactorily.

A successful development of a model requires adequate testing with regard to characteristics at different time scales and at a number of locations with different climates. For instance, a proper daily rainfall generation model should preserve the monthly and annual characteristics in addition to preserving various daily characteristics.

The aim of this study is to generate spatially averaged daily rainfall sequences using two daily rainfall generation models, namely, the Transition Probability Matrix (TPM) model and a modified Wang-Nathan Model (WNM) for Yarra region, one of the focused catchments in the CRC for Catchment Hydrology. Chapter 2 describes the screening of suitable stations for the study and preparation of a complete daily rainfall data set from 1955 to 1995. In Chapter 3, modeling algorithms of TPM and WNM models to generate daily rainfalls stochastically are described. The performance of the two models with respect to appropriate daily, monthly and annual rainfall characteristics is evaluated in Chapter 4. The conclusions drawn from the study are presented in Chapter 5.

2. DATA PREPARATION

2.1 Study Area

The study area is an approximate square of 128 km x 128 km that encompasses the entire Yarra catchment in Victoria. The area is demarcated by longitude from 144.76 to 146.22 degrees and latitude from -37.13 to -38.28 degrees. The Melbourne metropolitan area falls within the study area.

2.2 Selection of Rainfall Stations for the Study

Long daily rainfall records of good quality are required to calibrate and test rainfall generation models. The intention of the study was to select as many as stations with nearly complete rainfall records over a concurrent period of 40 years or more. The stations need to be selected to have a good spatial coverage and to be representative of different climatological regimes within the study area.

A dense network of daily rainfall stations is available for a significant proportion of the study area, in particular, in and around the Melbourne metropolitan area. There are 230 stations with daily rainfall records extending over 20 years and data available for at least 60% of the time. It was observed that rainfall records are more consistent and complete for the latter period from 1950 onwards.

Twenty rainfall stations were selected for this study having nearly complete records over the period from 1955 to 1995. The stations satisfy the following criteria.

- The accumulated and missing data together are less than 1% of the total data record for the selected period; data at many of the stations are almost complete.
- The selected stations provide a satisfactory spatial coverage over the study area.
- The 41 year period from 1955 to 1995 is the best compromise that guarantees concurrent and nearly complete data across the study area.

All the rainfall data required for this study were extracted from the ADAM database of the Bureau of Meteorology. The selected stations are listed in Table 1 and shown in Figure 1.

Table 1: Selected Stations and Thiessen Weights

CODE	STATION NAME	LATITUDE	LONGITUDE	HEIGHT (m)	THIESSEN WEIGHT
85085	TRAFALGAR	-38.2092	146.1533	50	0.097
86020	CHELTENHAM KINGSTON	-37.9564	145.0767	37	0.010
86027	CROYDON	-37.7897	145.2811	118	0.024
86070	MAROONDAH WEIR	-37.6497	145.5497	149	0.042
86071	MELBOURNE REGIONAL OFFICE	-37.8119	144.9664	31	0.034
86073	MICKLEHAM	-37.5547	144.8794	270	0.037
86074	MITCHAM	-37.8281	145.1900	159	0.014
86085	NARRE WARREN NORTH	-37.9908	145.3342	120	0.091
86090	O'SHANNASSY	-37.7089	145.7856	240	0.135
86096	PRESTON RESERVOIR	-37.7267	145.0050	94	0.022
86106	SILVAN	-37.8331	145.4331	259	0.044
86111	SPRINGVALE NECROPOLIS	-37.9467	145.1747	39	0.018
86117	TOOROURRONG RESERVOIR	-37.4769	145.1517	244	0.012
86125	WHITTLESEA	-37.5042	145.1242	198	0.020
86146	BEAUMARIS	-37.9783	145.0258	30	0.073
88000	ALEXANDRA (ACHERON)	-37.2528	145.7167	180	0.055
88023	LAKE EILDON	-37.2325	145.9108	262	0.091
88034	KILMORE ASSUMPTION	-37.3011	144.9475	370	0.061
88060	WALLABY CREEK WEIR	-37.4489	145.2144	520	0.071
88131	NARBETHONG	-37.5017	145.6781	340	0.049

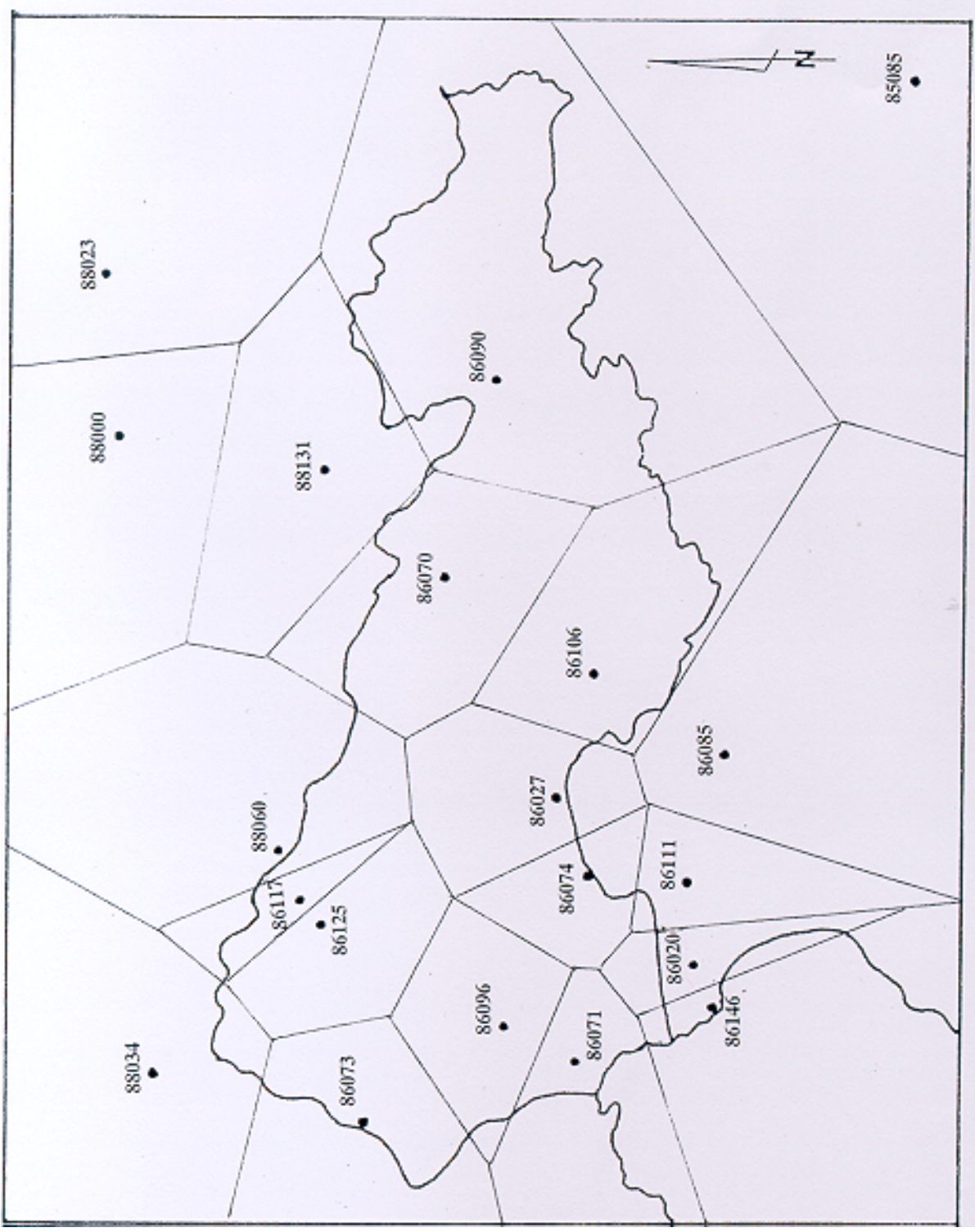


Figure 1: Rainfall stations in the vicinity of Yarra Catchment and constructed Thiessen Polygons for computing areal average

2.3 Missing and Accumulated data

Missing daily data were filled-in with corresponding rainfall data at the nearest station. These include a few whole months with no data. Accumulated data are usually in the durations of two to three days; these were uniformly distributed over the respective accumulated period.

As the missing and accumulated data are only a very small proportion of the dataset (< 1%), the uncertainties associated with the approximate procedures adopted to fill-in missing data and disaggregate accumulated data are considered to be very small and would not cause any significant effect on the results obtained based on these data.

2.4 Derivation of Areal Average Daily Rainfall Series

A Thiessen weighted daily average rainfall series (1955-1995) was derived for the Yarra catchment area using the rainfall data at the 20 selected stations. The constructed Thiessen polygons are shown Figure 1 and the Thiessen weights for respective stations are given in Table 1. It should be noted that, as the study area is quite large and encompasses a range of different climatological regimes, the resultant areal rainfall series tends to be more uniform and would generally be different from the representative characteristics of point rainfall series of constituent gauging stations.

This areal rainfall series is used to generate spatially averaged daily rainfalls over the study area, and then to evaluate the performance of the generating models by comparing various statistics of the generated and historic series.

3. GENERATION OF AREAL AVERAGE RAINFALLS

This study uses two daily rainfall generation models, namely, Transition Probability Matrix (TPM) model and modified Wang-Natham Model (WNM) to generate synthetic sequences of daily rainfalls for the study area. The models can be used to generate a large number of replicates that are equally likely to occur; the performance of the two models then can be evaluated by comparing various statistical properties of the generated sequences with the properties of the historical sequence.

This report is confined to the evaluation of the models with respect to generation of spatially averaged daily rainfalls. This is carried out by using the areal average daily rainfall series derived in Section 2.4 to generate synthetic sequences and then the appropriate statistical properties of the generated series are used to evaluate the models.

3.1 Description of Generation Models

3.1.1 Transition Probability Matrix (TPM) Model

The seasonality in occurrence and magnitude of daily rainfall is taken into account by considering each month separately. The daily rainfall is divided into a number of states up to a maximum of seven. State 1 is dry (no rainfall) and the other states are wet. The shifted Gamma distribution is used for the largest state while a linear distribution is used for the intermediate states. The latter is chosen because daily rainfall usually exhibits a J shape distribution.

The transition probabilities are estimated from

$$p_{ij}(k) = f_{ij}(k) / \sum_{j=1}^C f_{ij}(k) \quad i, j = 1, 2, \dots, C; k = 1, 2, \dots, 12 \quad (3.1)$$

where $f_{ij}(k)$ = historical frequency of transition from state i to state j within month k , and
 C = the maximum number of states.

The Gamma distribution parameters are obtained by the method of moments.

The daily rainfall data are generated by following the steps set out below assuming that the initial state is dry (that is, state one).

- Step 1: Generate a uniformly distributed random number U between 0 and 1. Using the appropriate TPM for the month, determine the state of the next day.
- Step 2: If the state is wet, go to step 3. Otherwise, set the rainfall depth to zero and go to step 1.
- Step 3: Calculate the rainfall depth by using the linear distribution for the intermediate states and shifted Gamma distribution for the largest state.
- Step 4: Repeat steps 1 to 3 until the required length of daily rainfall data is required.

The generating model is improved by adopting an empirical adjustment factor (F) to match the observed standard deviation of the annual rainfall (Boughton, 1999). The adjustment factor is obtained by trial and error until the frequency distribution of the observed and generated annual rainfalls matches. The generated daily rainfall in each year is multiplied by the following ratio:

$$\text{Ratio}_i = \{M + (T_i - M)F\} / T_i \quad (3.2)$$

where M = the observed mean annual rainfall, and
 T_i = the generated annual rainfall for year i .

Since the slope of the frequency curve is proportional to the standard deviation, the adjustment factor can be directly obtained as a ratio of the standard deviation of the observed and generated annual rainfall. Thus:

$$F = \text{stdev}_g / \text{stdev}_o \quad (3.3)$$

The adjusted annual total is obtained from

$$T_i' = G + (T_i - G)F \quad (3.4)$$

where G is the generated mean annual rainfall.

By dividing both sides of Eq (3.4) by T_i , we obtain the ratio of the adjusted annual rainfall to the unadjusted generated annual rainfall.

$$T_i' / T_i = \{G + (T_i - G)F\} / T_i \quad (3.5)$$

Eq (3.5) is identical to Eq (3.2) except that the observed mean (M) in Eq (3.2) is replaced by G in Eq (3.5). This minimises the bias in the mean rainfall.

The standard deviation of the generated annual rainfall is estimated from a number of replicates and averaged. The ratio of this adjusted value to the observed value is taken as F for adjusting the daily values.

3.1.2 Modified Wang-Nathan Model (WNM)

The occurrence of rainfall is determined by using a first order Markov chain using the two transition probabilities: $p_{W|D}$, the conditional probability of a wet day given that the previous day was dry; $p_{W|W}$, the conditional probability of a wet day given that the previous day was wet. The unconditional probability of a wet day can be derived as

$$\pi = \frac{P_{W|D}}{1 + P_{W|D} - P_{W|W}} \quad (3.6)$$

The rainfall depth is obtained from a Gamma distribution whose probability density function is given by

$$f(x) = \frac{(x/\beta)^{\alpha-1} \exp(-x/\beta)}{\beta\Gamma(\alpha)} \quad (3.7)$$

where α is the shape parameter and β the scale parameter. The mean and variance of the Gamma distribution are given by

$$\mu(x) = \alpha\beta \quad (3.8)$$

$$\sigma^2(x) = \alpha\beta^2 \quad (3.9)$$

The mean and variance of the rainfall total, X, over a month of N days is given by (Katz, 1983, 1985)

$$\mu(X) = N\pi\alpha\beta \quad (3.10)$$

$$\sigma^2(X) = N\pi\alpha\beta^2 \left[1 + \alpha(1 - \pi) \frac{1 + P_{W|W} - P_{W|D}}{1 - P_{W|W} + P_{W|D}} \right] \quad (3.11)$$

The modified Wang-Nathan model involves the following steps.

- Step 1: For month k, generate a sequence of wet and dry days for the whole month using a two-state first order Markov chain.
- Step 2: For any wet day in that month, generate a daily rainfall amount x^d from a Gamma distribution with parameters $\alpha = \alpha^d$ and $\beta = \beta^d$ which are estimated from the mean and variance of daily rainfall amounts by using Eq (3.8) and (3.9).
- Step 3: Manipulate the monthly total of the daily rainfall generated in step 2, $\tilde{X}_i = \sum x^d$, to produce a new monthly total X_i by using the Thomas-Fiering monthly model.

$$\frac{X_i - \mu(X_i)}{\sigma(X_i)} = \rho \frac{X_{i-1} - \mu(X_{i-1})}{\sigma(X_{i-1})} + (1 - \rho^2)^{1/2} \frac{\tilde{X}_i - \mu'(X_i)}{\sigma'(X_i)} \quad (3.12)$$

The subscripts i-1 and i in Eq (3.12) denote the previous and current months respectively. The lag one autocorrelation is estimated from the non-seasonal data.

The mean $\mu'(X_i)$ and standard deviation $\sigma'(X_i)$ used in Eq (3.12) are obtained from Eq (3.10) and (3.11) using the daily Gamma parameters obtained in step 2.

- Step 4: Produce a new daily rainfall series x for that month by multiplying all the x^d by a factor $(X_i/\sum x^d)$.

The modified Wang-Nathan model (WNM) generates only one sequence of daily rainfall amounts, in comparison to the original model, but at the same time adjusts the daily rainfall to match the monthly characteristics.

4. EVALUATION OF GENERATION MODELS

One hundred replicates with the length of replicate equal to the historical data (1955-1995) were generated using the above two models. The replicates represent one hundred synthesized areal daily rainfall series for the study area that are equally likely to occur under prevailing conditions.

The models are then evaluated based on their capability to preserve various statistical properties of the historical data. A successful model should preserve the monthly and annual characteristics in addition to preserving various daily characteristics. The following statistical parameters from the generated replicates and historical data were used to evaluate the generated data.

Annual statistics

- Mean annual rainfall
- Standard deviation of annual rainfall
- Coefficient of skewness of annual rainfall
- Serial correlation
- Maximum
- Minimum
- Range
- 2-year, 3-year, 5-year, 7-year, 10-year low rainfall sums

Monthly statistics (for each month of the year)

- Mean monthly rainfall
- Standard deviation of monthly rainfall
- Skewness of monthly rainfall
- Serial correlation
- Minimum
- Maximum
- Number of wet days during the month

Daily statistics

- Mean daily rainfall
- Standard deviation of daily rainfall
- Skewness of daily rainfall
- Mean daily rainfall for solitary wet days (WET 1)
- Mean daily rainfall for wet days bounded only on one side by a wet day (WET 2)
- Mean daily rainfall for wet days bounded on both sides by wet days (WET 3)
- Correlation between rainfall depth and duration
- Mean dry spell (days)
- Standard deviation of dry spell (days)
- Skewness of dry spell
- Mean wet spell (mean)
- Standard deviation of wet spell (mean)
- Skewness of wet spell

The statistics derived from the generated rainfall replicates using the TPM and WNM models are given in Appendices A (Tables A.1 and A.2) and B (Tables B.1 and B.2) respectively. The mean, standard deviation, minimum, maximum, 25% and 75% percentiles for each parameter is presented to indicate the variability within the 100 estimates. The corresponding historic values are also given in these tables.

The annual rainfall statistics (mean, standard deviation, skew and serial correlation) derived from the replicates generated using the TPM and WNM models are compared against corresponding historic parameters in Figures A.1 and B.1 respectively (Appendices A and B). Similar plots for monthly and daily statistics are shown in Figures A.2, A.3, B.2 and B.3. These plots provide a consistent basis for evaluation of the performance of the generation models with respect to preserving the historic characteristics. The plots present the mean, 25% and 75% percentiles and the range (maximum and minimum) of the respective rainfall characteristic, derived from the 100 replicates of the generated rainfall sequences for each month. The relative position of the respective historic values within the spectrum of variability of generated estimates indicates the capability of the generating model to preserve the rainfall characteristics of interest.

The generating model is considered performing well if the generated and historic means are close to each other. If the historic mean consistently lies within the 25% to 75% percentiles of generated values, the model is generally considered to be performing satisfactorily. The plots are also indicative of any persistent biases in the generating algorithm, such as generated mean consistently being lower or higher than the historical mean throughout the year.

The performance of the two models with respect to different daily, monthly and annual characteristics are summarized in Table 2.

Table 2(a): Evaluation of Annual Statistics

Property	TPM Model	WNM Model
Mean annual rainfall	Poorly modelled; generated mean is 3.4% higher than the historic mean; historic mean falls below 25% percentile of the generated values.	Well modelled; generated mean is very close to historic mean.
Standard deviation of annual rainfall	Well modelled.	Poorly modelled; generated value is 13% higher; historic value lies above 75% of the generated values.
Skew of annual rainfall	Poorly modelled. Generated rainfall is positively skewed (+0.85) whereas the historic rainfall is negatively skewed (-0.25).	Reasonably modelled; historic value lies within middle 50% of the generated values.
Serial correlation coefficient (annual series)	Modelling is not satisfactory; average serial correlation coefficient of the generated series is 0.08; in contrast, historic series exhibits a weak negative correlation (-0.17).	Modelling is not satisfactory; the generated series appear to be uncorrelated ($R=0.02$), whereas, historic series exhibits a weak negative correlation (-0.17).

Table 2(b): Evaluation of Monthly Statistics

Property	TPM Model	WNM Model
Mean monthly rainfall	Reasonably modelled; historic mean is consistently lower than the generated mean for all months but lies within the range of 25-50% of the generated values. (11/12)	Very well modelled; generated rainfalls are very close to historic values. (12/12)
Standard deviation of monthly rainfall	Reasonably modelled; for 8 months, the historic mean lies within middle 50% of the generated values. (8/12)	Very well modelled; generated values are very close to historic values. (12/12)
Skew of monthly rainfall	Poorly modelled; only for 4 months, the historic values lie well within middle 50% of the generated values; in general, skew of the generated rainfalls is higher. (4/12)	Reasonably modelled; for 8 months, historic values lie within middle 50% of generated values. (8/12)
Serial correlation coefficient (monthly series)	Not satisfactorily modeled; Generated monthly series appear to be serially uncorrelated ($R < 0.07$), whereas, serial correlation of the historic series exhibits greater monthly variation from -0.14 (Feb) to 0.38 (May). (6/12)	Not satisfactorily modeled; the serial correlation coefficient of the generated series is consistently around 0.10 for all months, whereas, serial correlation of the historic series exhibits greater monthly variation from -0.14 (Feb) to 0.38 (May). (7/12)

() indicates number of months for which historic values lie within middle 50% of the generated values

Table 2(c): Evaluation of Daily Statistics

Property	TPM Model	WNM Model
Mean daily rainfall	Poorly modelled; the historic rainfalls are consistently lower and generally lie within lowest 25% of the generated rainfalls. (0/12)	Well modelled; generated values are generally very close to historic values. (12/12)
Standard deviation of daily rainfall	Well modelled; historic values are well within the middle 50% of the generated values. (12/12)	Well modelled; historic values are within the middle 50% of the generated values, except for one month. (11/12)
Skew of daily rainfall	Well modelled; generated values are close to historic values except for February (11/12)	Reasonably modelled; for 7 months, historic values lie within middle 50% of generated values. (7/12)
WET 1	Poorly modelled; generated values are consistently higher, except from January to March. (5/12)	Very poorly modelled; generated values are exceptionally higher; historic values lie below the minimum of generated values (0/12)
WET 2	Poorly modelled; historic values generally lie within lowest 10% of the generated values. (0/12)	Very poorly modelled; historic values lie below the minimum of the generated values. (0/12)
WET 3	Reasonably modelled; generated values are slightly higher; historic values generally lie within 25-50% of the generated values. (0/12)	Very poorly modelled; historic values lie above the maximum of the generated values. (0/12)
Dry Spell- mean	Satisfactorily modelled; historic values generally lie within middle 50% of the generated values. (12/12)	Same as for the TPM model. (12/12)
Dry Spell- standard deviation	Reasonably modelled; generated values tend to be slightly higher than the historic values. (7/12)	Same as for the TPM model. (8/12)
Dry Spell- skew	Poorly modelled for June, July and November and generally satisfactory for other months; generated values are consistently higher than the historic values. (9/12)	Same as for the TPM model (9/12)
Wet Spell- mean	Poorly modelled from June to September (well below the historic values); modelling for remaining months is acceptable. (8/12)	Satisfactorily modelled except for June and July. (9/12)
Wet Spell- standard deviation	Poorly modelled from May to September; remaining months are reasonably modelled. (7/12)	Poorly modelled from May to July; modeling for remaining months are generally acceptable (7/12)
Wet Spell- skew	Generated values are consistently lower; historic values generally lie above 75% percentile of the generated values. (3/12)	Modelling is generally satisfactory. (11/12)
Number of wet days for month	Generated values are slightly but consistently lower than the historic values. (4/12)	Very well modeled. (11/12)

() indicates number of months for which historic values lie within middle 50% of the generated values

The WNM model incorporates some adjustments to the generated monthly mean and standard deviation, in order to guarantee that the mean is satisfactorily modelled. Likewise, in the TPM model, some adjustments are made to match the standard deviation of the annual historic data. Generally, it would not be possible to model satisfactorily all the statistics at various time scales, but it is imperative that important parameters such as the mean and standard deviation of generated rainfalls are satisfactorily modelled at daily, monthly and annual time scales.

It could be concluded that the WNM model outperforms the TPM model with respect to preserving important statistics such as the mean, standard deviation and skew. The TPM model grossly overestimates the historic mean at daily, monthly and yearly time scales, whereas, WNM's performance is nearly perfect. The WNM model also preserves standard deviation and skew of the rainfalls satisfactorily, in comparison to the TPM model. However, the TPM model performs better with respect to standard deviation on annual scale. Modelling of number of wet days is also much superior with the WNM model.

Both models show some deficiency in preserving the annual and monthly serial correlation. The historic series indicates an appreciable serial correlation for May with correlation coefficient (R) of 0.38 and a weak serial correlation from July to December with R ranging from 0.12 to 0.25. The serial correlation of the generated series does not change adequately over the year and consistently remain the same. The performance of the WNM model is slightly better than that of the TPM model.

With respect to modeling of dry spell and wet spell characteristics, the performance of the WNM model is either as good as TPM or slightly better. Performance of both models is generally satisfactory. Modelling of WET 1, WET 2 and WET 2 statistics is of some concern, as none of the models are capable of modeling these characteristics adequately, in particular, the performance of the WNM model is exceptionally poor.

Overall, it could be concluded that the modified Wang-Nathan Model is the better daily rainfall generation model, based on limited testing carried out during this study.

5. CONCLUSION

This study concentrated on generating areal average daily rainfalls for Yarra region in Victoria using two stochastic rainfall generation models, Transition Probability Matrix Model and modified Wang-Nathan Model. The following conclusions can be drawn from the results of this study:

- The performance of the WNM daily rainfall generation model is superior to TPM model with respect to preserving the mean, standard deviation and skew of the historic rainfall data at daily, monthly and annual time scales.
- Dry spell and wet spell characteristics are modelled satisfactorily by both models; the performance of WNM model is slightly better.

- Both models are incapable of preserving the monthly variation of serial correlation. The WNM model performs slightly better.
- WET 1, WET 2 and WET 3 characteristics are not modelled satisfactorily by either model; the performance of the WNM model is poorer.

The modified Wang-Nathan Model was shown to be the superior rainfall generation model; however, the results of this study should be viewed with some caution, as they are based on testing only one data set.

6. REFERENCES

Boughton W.C., 1999. A daily rainfall generating model for water yield and flood studies. CRC for Catchment Hydrology, Report 99/9, pp21.

Chapman, T., 1994. Stochastic models for daily rainfall. The Institution of Engineers Australia, National Conference Publication 94/15, pp.7-12.

Katz, R. W. 1983. Statistical procedures for making inferences about precipitation changes simulated by an atmospheric general circulation model. *Journal of Atmospheric Sciences*, 40, 2193-2201.

Katz, R. W. 1985. Probabilistic models. In *Probability, statistics and decision making in the atmospheric sciences*, edited by A. H. Murphy and R. W. Katz, Westview, 251-288.

Srikanthan, R. and McMahon, T.A., 1985. Stochastic generation of rainfall and evaporation data. Australian Water Resources Council, Tech. Paper. No. 84, AGPS, Canberra, 301pp.

APPENDIX A

TRANSITION PROBABILITY MATRIX MODEL RESULTS

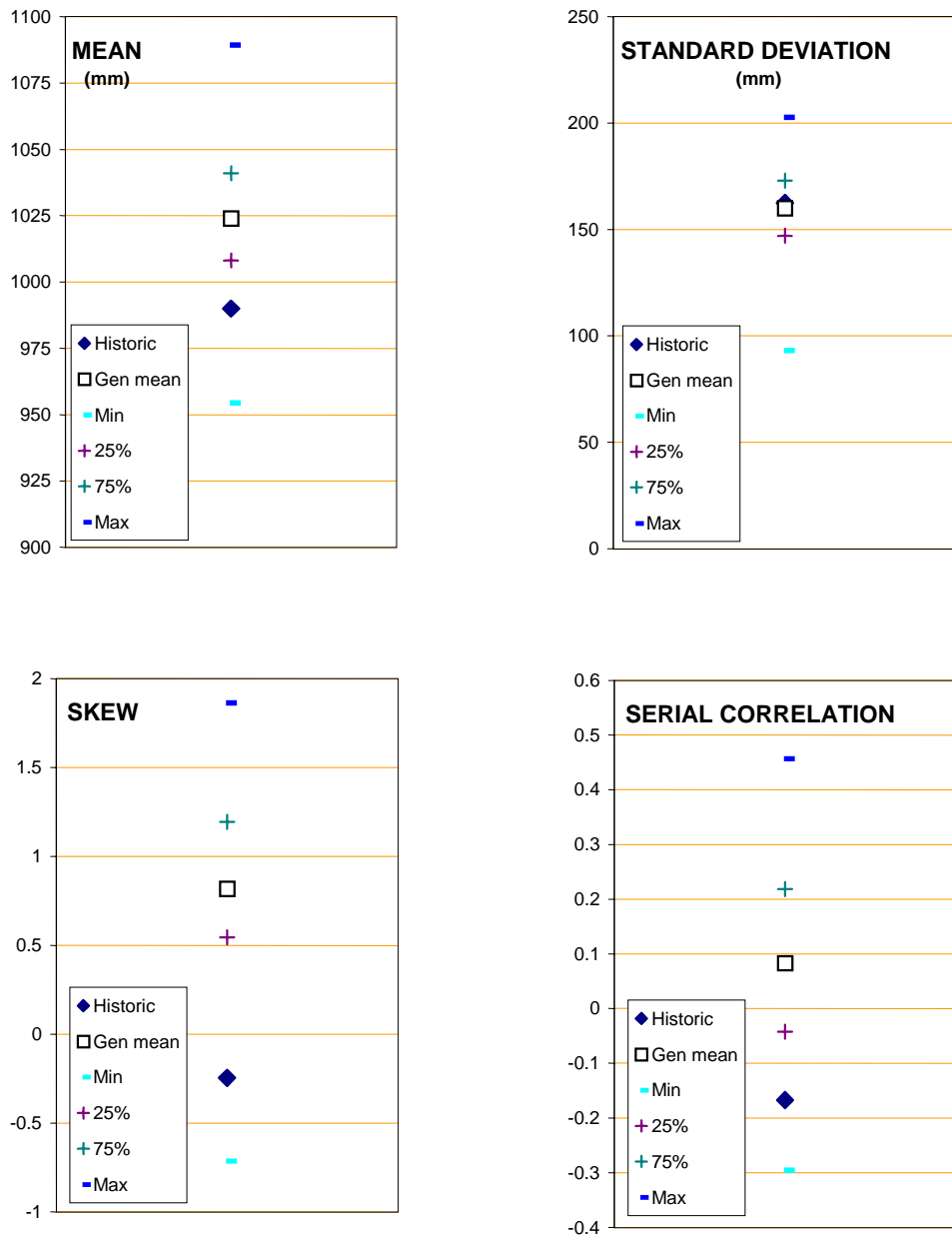


Figure A.1: Evaluation of annual rainfall statistics for Transition Probability Matrix Model (mean, standard deviation, skew and serial correlation)

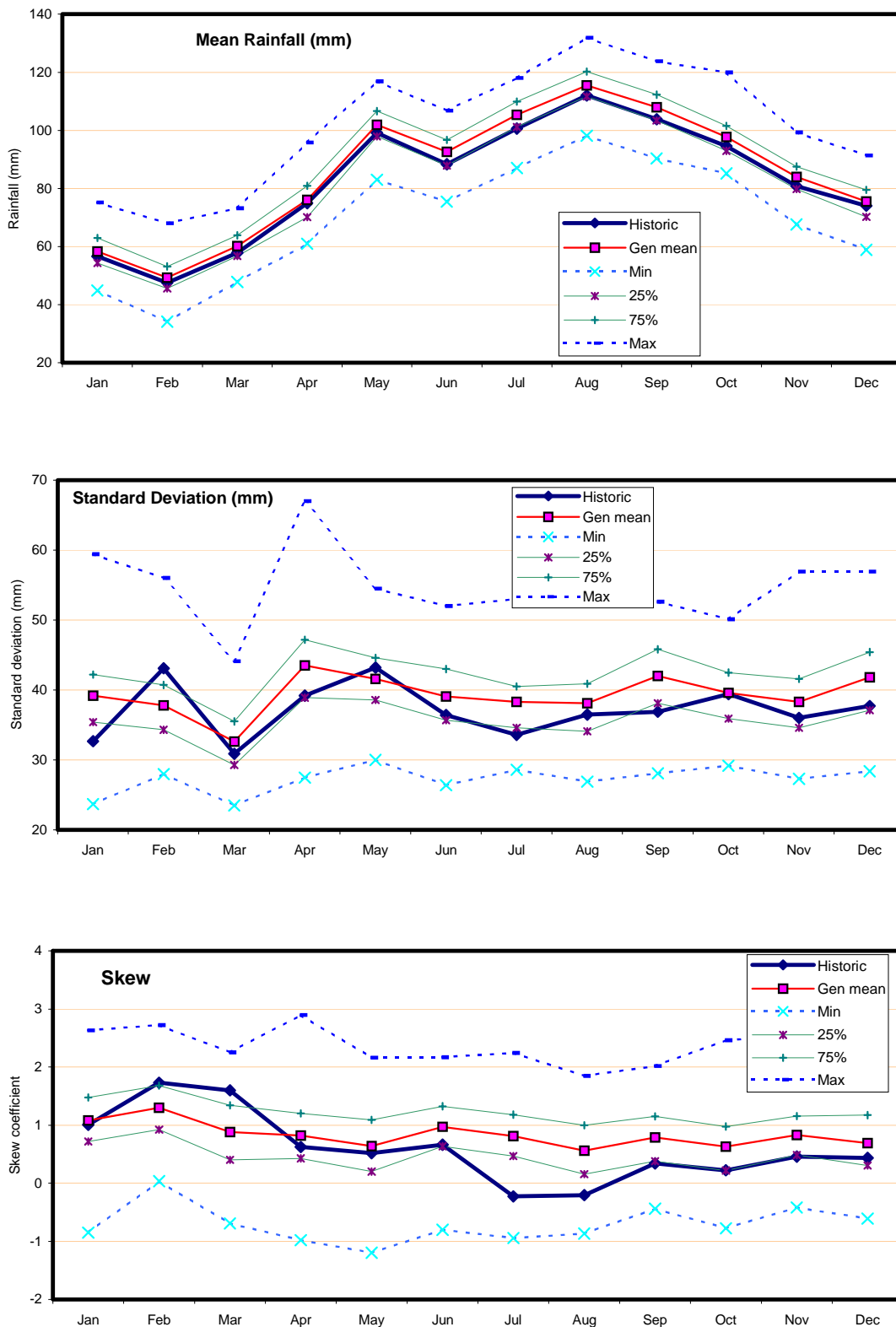


Figure A.2(a) : Evaluation of monthly rainfall statistics (mean, standard deviation, skew) Transition Probability Matrix Model

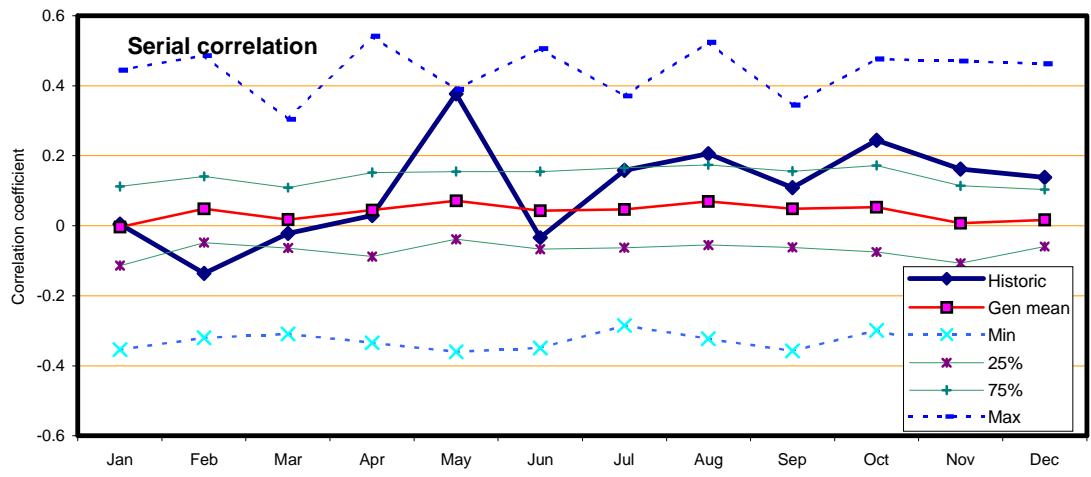


Figure A.2(b) : Evaluation of monthly rainfall statistics (serial correlation)
Transition Probability Matrix Model

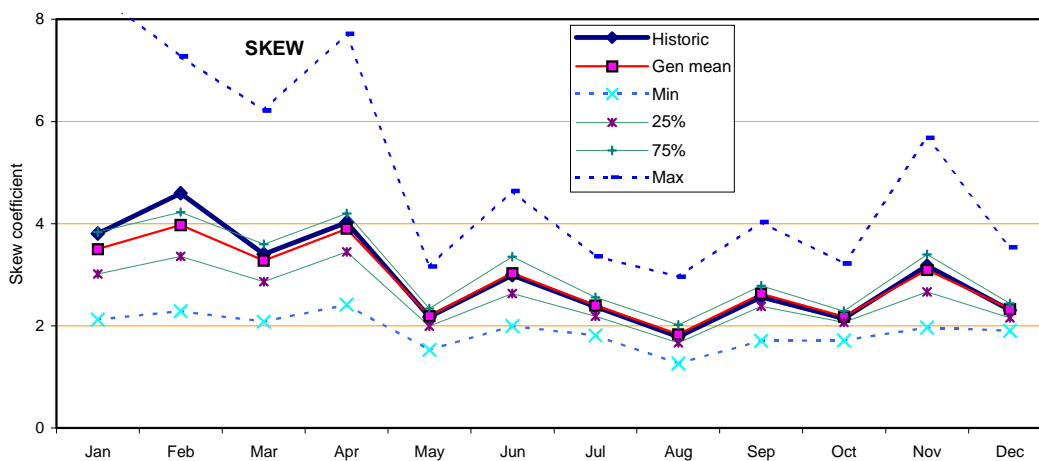
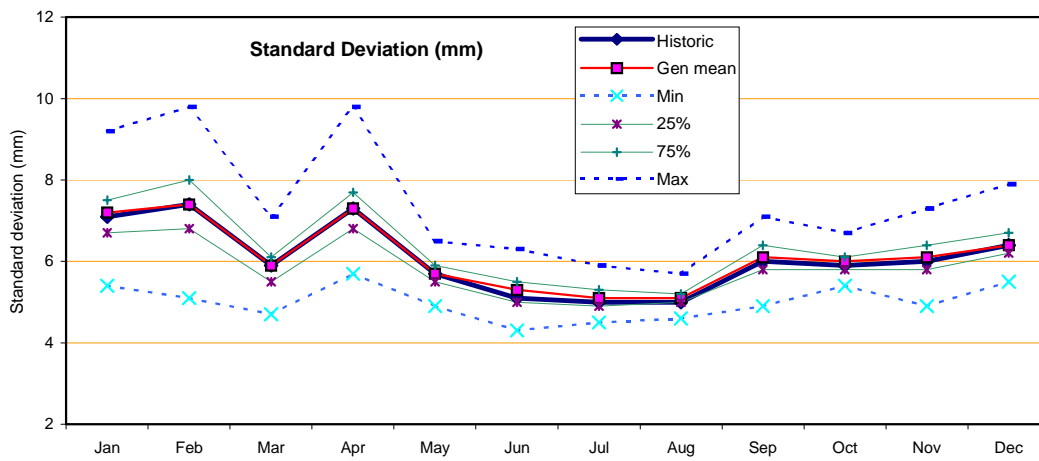
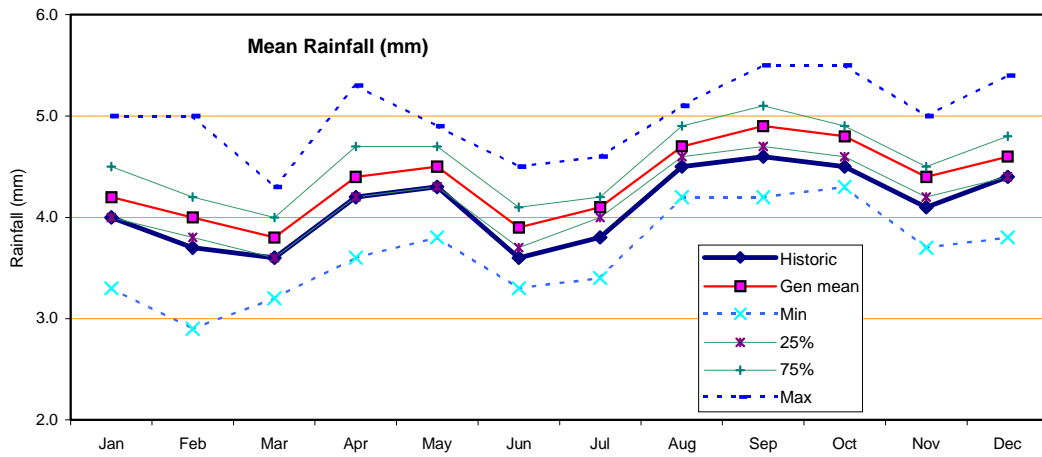


Figure A.3(a): Evaluation of daily rainfall statistics (mean, standard deviation, skew Transition Probability Matrix Model

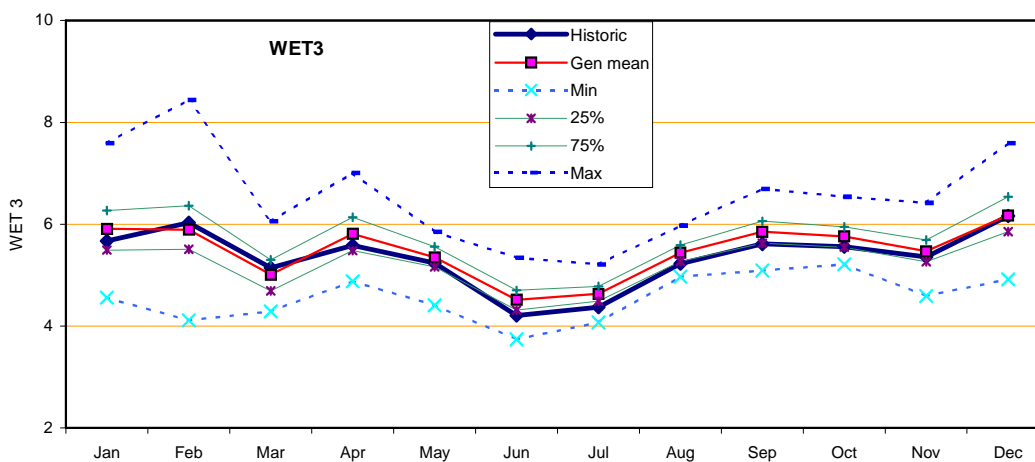
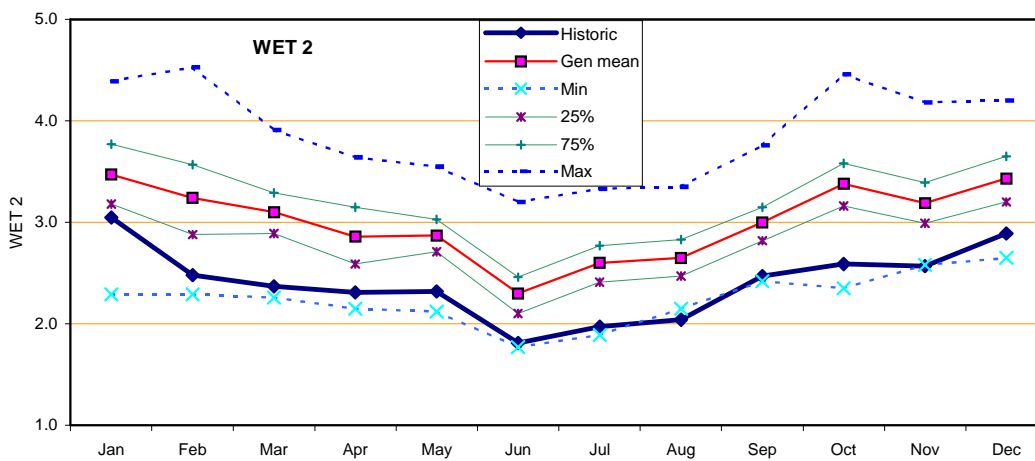
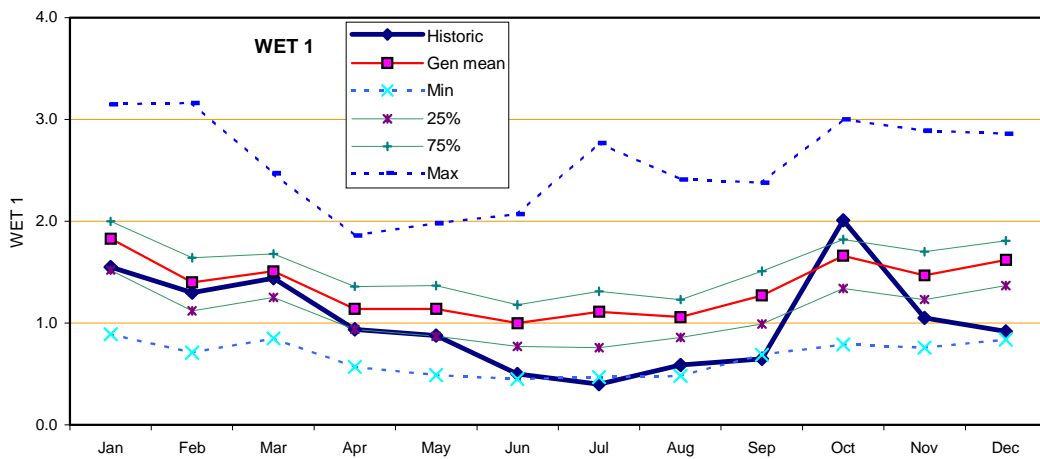


Figure A.3(b): Evaluation of daily rainfall statistics (WET 1,WET 2,WET 3)
Transition Probability Matrix Model

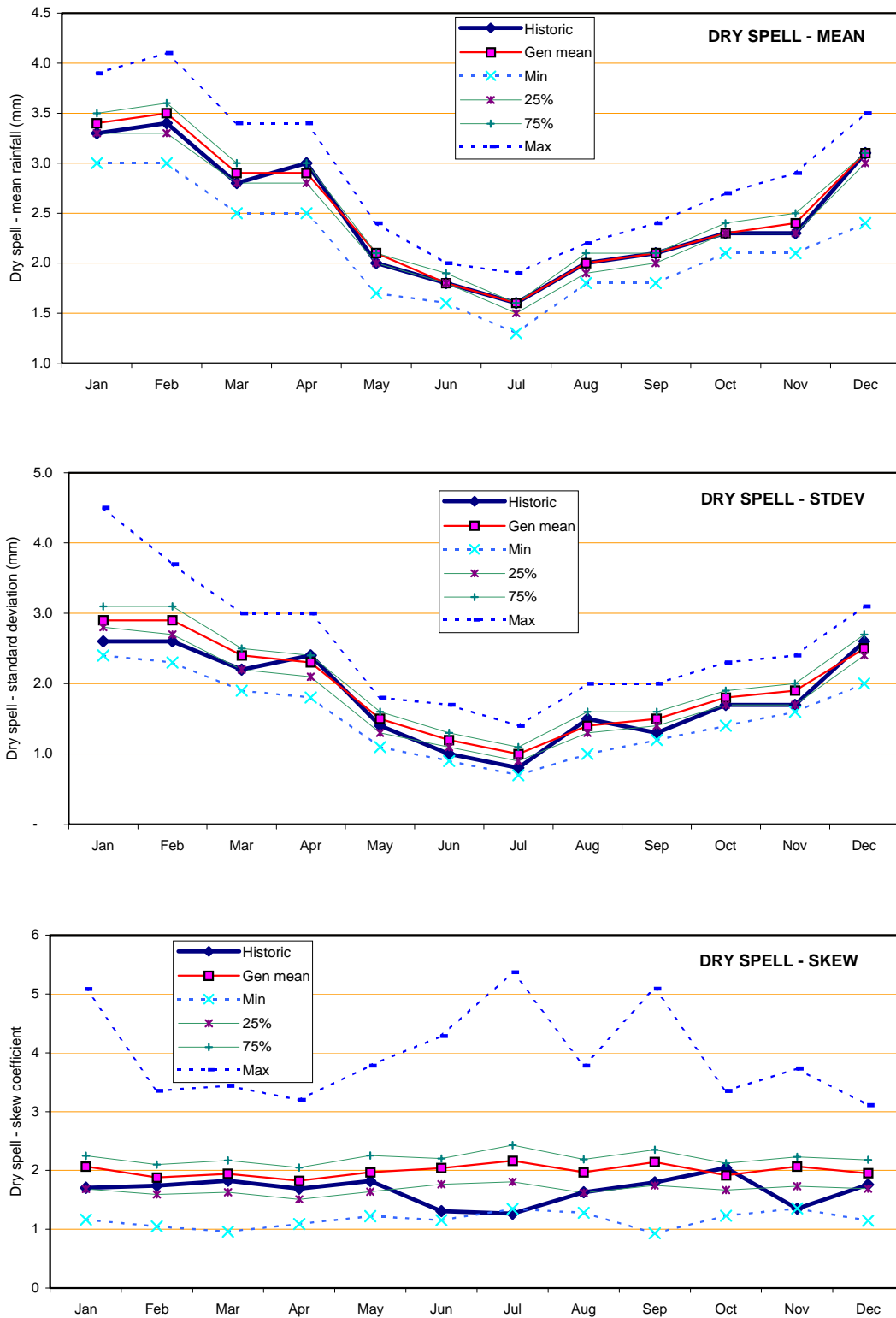


Figure A.3(c): Evaluation of daily rainfall statistics (dry spell mean, standard deviation, skew) Transition Probability Matrix Model

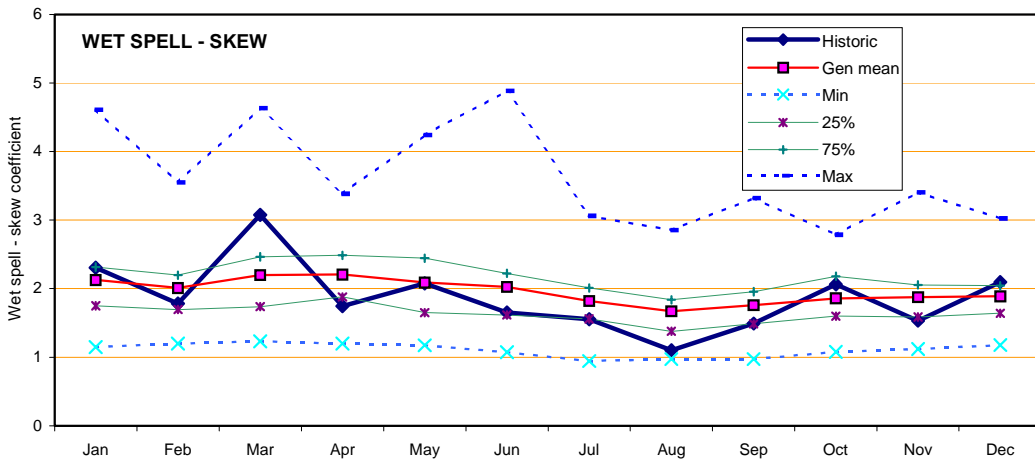
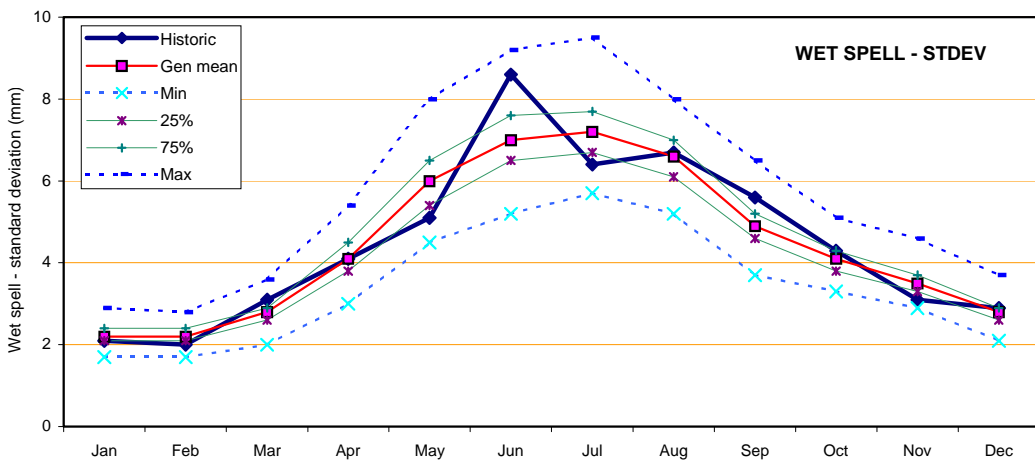
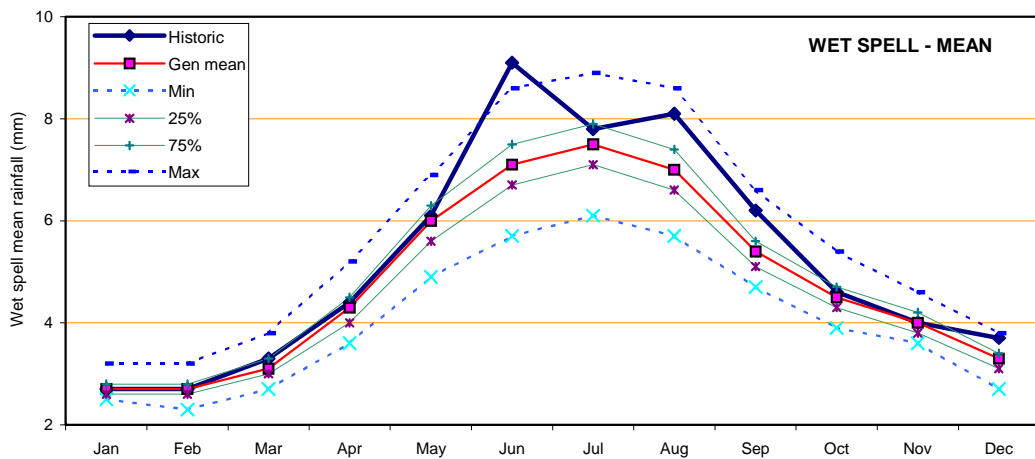


Figure A.3(d): Evaluation of daily rainfall statistics (wet spell mean, standard deviation, skew) Transition Probability Matrix Model

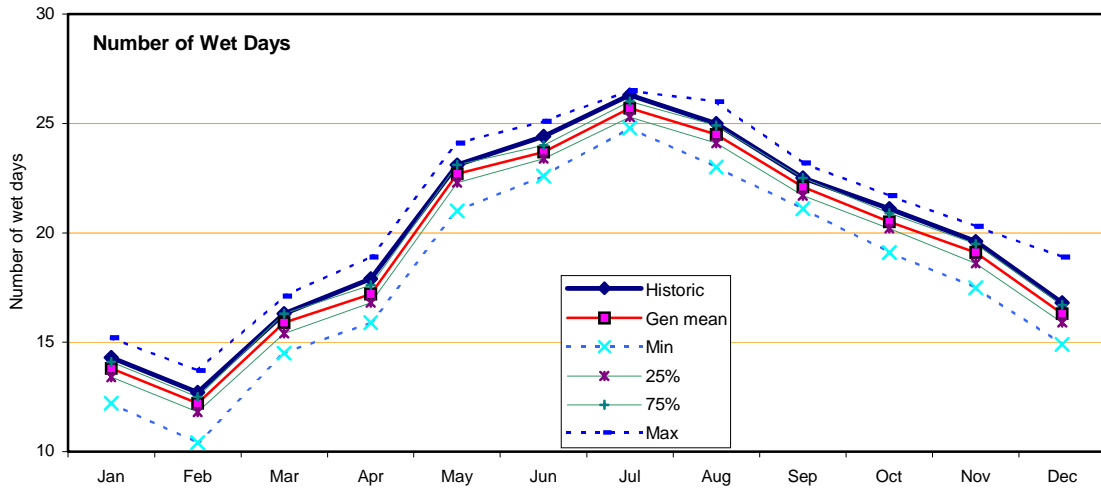


Figure A.3(e): Evaluation of daily rainfall statistics (number of wet days)
Transition Probability Matrix Model

Table A.1: Annual and monthly statistics for generated rainfall using TPM Model

ANNUAL STATISTICS

	Annual	StDev	Skew	R1	Min	Max	Range	2-year	3-year	5-year	7-year	10-year
Historic	990	162	-0.25	-0.17	0.64	1.33	1.00	1.62	2.55	4.37	6.24	9.19
Gen mean	1024	160	0.82	0.08	0.71	1.41	1.95	1.61	2.57	4.51	6.52	9.57
StDev	26	19	0.52	0.17	0.07	0.08	0.74	0.10	0.12	0.19	0.27	0.34
Min	955	93	-0.71	-0.30	0.55	1.19	0.63	1.35	2.28	3.86	5.43	8.42
25%	1008	147	0.55	-0.04	0.66	1.36	1.34	1.55	2.49	4.38	6.34	9.37
Med	1025	161	0.86	0.09	0.72	1.40	1.90	1.61	2.57	4.53	6.55	9.61
75%	1041	173	1.20	0.22	0.76	1.45	2.49	1.68	2.65	4.64	6.70	9.79
Max	1089	203	1.86	0.46	0.86	1.71	3.85	1.81	2.86	4.93	7.11	10.41

MONTHLY STATISTICS

Mean monthly rainfall (mm)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	56.6	47.5	57.8	74.9	99.1	88.3	100.6	112.0	103.8	94.6	80.8	74.0
Gen mean	58.3	49.3	60.2	76.0	101.9	92.6	105.3	115.5	107.9	97.7	84.0	75.5
StDev	5.8	5.9	5.3	7.5	7.1	6.1	5.9	6.3	6.6	6.3	6.1	6.5
Min	44.8	34.1	47.8	61.0	83.0	75.4	87.0	98.2	90.3	85.2	67.6	58.9
25%	54.3	45.5	56.7	70.1	97.9	87.9	101.1	111.7	103.4	92.9	79.8	70.2
Med	57.9	48.7	60.3	75.1	101.4	93.5	105.0	115.5	106.7	97.0	84.6	76.5
75%	63.0	53.1	63.8	80.9	106.6	96.7	109.9	120.2	112.3	101.6	87.5	79.5
Max	75.2	68.0	73.2	95.8	116.9	106.8	118.0	131.9	123.8	120.0	99.3	91.4

Standard Deviation (monthly rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	32.7	43.1	30.9	39.2	43.2	36.4	33.6	36.5	36.9	39.4	36.0	37.7
Gen mean	39.2	37.8	32.6	43.5	41.6	39.1	38.3	38.1	42.0	39.6	38.3	41.8
StDev	6.2	5.5	4.6	6.5	5.0	5.9	5.0	4.7	5.1	4.4	5.5	6.3
Min	23.7	28.0	23.5	27.5	30.0	26.4	28.6	26.9	28.1	29.2	27.3	28.4
25%	35.4	34.3	29.3	38.9	38.6	35.7	34.6	34.1	38.1	35.9	34.6	37.1
Med	38.6	37.4	31.9	42.9	41.6	39.6	37.5	38.1	41.9	39.9	38.4	41.2
75%	42.2	40.7	35.5	47.2	44.6	43.0	40.5	40.9	45.8	42.5	41.6	45.4
Max	59.4	56.0	44.1	67.0	54.5	52.0	53.0	52.5	52.6	50.1	56.9	56.9

Skew coefficient (monthly rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.01	1.73	1.60	0.62	0.52	0.66	-0.23	-0.21	0.34	0.22	0.46	0.44
Gen mean	1.08	1.30	0.88	0.82	0.64	0.97	0.81	0.56	0.79	0.63	0.83	0.69
StDev	0.60	0.56	0.64	0.66	0.68	0.54	0.54	0.58	0.53	0.54	0.56	0.61
Min	-0.85	0.03	-0.69	-0.98	-1.20	-0.80	-0.95	-0.87	-0.44	-0.78	-0.42	-0.61
25%	0.72	0.92	0.40	0.43	0.20	0.64	0.47	0.16	0.38	0.22	0.49	0.30
Med	1.09	1.28	0.89	0.88	0.70	1.08	0.85	0.64	0.80	0.64	0.82	0.67
75%	1.48	1.68	1.34	1.20	1.09	1.33	1.18	1.00	1.15	0.98	1.15	1.17
Max	2.63	2.72	2.25	2.90	2.16	2.17	2.25	1.85	2.02	2.46	2.54	2.15

Serial correlation (monthly rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	0.00	-0.14	-0.02	0.03	0.38	-0.03	0.16	0.21	0.11	0.24	0.16	0.14
Gen mean	0.00	0.05	0.02	0.04	0.07	0.04	0.05	0.07	0.05	0.05	0.01	0.02
StDev	0.16	0.15	0.13	0.17	0.14	0.16	0.15	0.17	0.16	0.17	0.16	0.15
Min	-0.35	-0.32	-0.31	-0.33	-0.36	-0.35	-0.29	-0.32	-0.36	-0.30	-0.35	-0.49
25%	-0.11	-0.05	-0.06	-0.09	-0.04	-0.07	-0.06	-0.05	-0.06	-0.08	-0.11	-0.06
Med	-0.02	0.05	0.02	0.04	0.09	0.06	0.04	0.09	0.04	0.04	0.01	0.02
75%	0.11	0.14	0.11	0.15	0.16	0.15	0.17	0.17	0.16	0.17	0.11	0.10
Max	0.44	0.49	0.30	0.54	0.39	0.51	0.37	0.52	0.35	0.48	0.47	0.46

Table A.2 : Statistics for generated daily rainfall using Transition Probability Matrix Model

Mean daily rainfall (mm)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	4.0	3.7	3.6	4.2	4.3	3.6	3.8	4.5	4.6	4.5	4.1	4.4
Gen mean	4.2	4.0	3.8	4.4	4.5	3.9	4.1	4.7	4.9	4.8	4.4	4.6
StDev	0.3	0.4	0.3	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3
Min	3.3	2.9	3.2	3.6	3.8	3.3	3.4	4.2	4.2	4.3	3.7	3.8
25%	4.0	3.8	3.6	4.2	4.3	3.7	4.0	4.6	4.7	4.6	4.2	4.4
Med	4.2	4.1	3.8	4.3	4.5	3.9	4.1	4.7	4.9	4.7	4.4	4.6
75%	4.5	4.2	4.0	4.7	4.7	4.1	4.2	4.9	5.1	4.9	4.5	4.8
Max	5.0	5.0	4.3	5.3	4.9	4.5	4.6	5.1	5.5	5.5	5.0	5.4

Standard Deviation (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	7.1	7.4	5.9	7.3	5.7	5.1	5.0	5.0	6.0	5.9	6.0	6.4
Gen mean	7.2	7.4	5.9	7.3	5.7	5.3	5.1	5.1	6.1	6.0	6.1	6.4
StDev	0.8	0.9	0.5	0.8	0.3	0.4	0.3	0.2	0.4	0.3	0.5	0.4
Min	5.4	5.1	4.7	5.7	4.9	4.3	4.5	4.6	4.9	5.4	4.9	5.5
25%	6.7	6.8	5.5	6.8	5.5	5.0	4.9	5.0	5.8	5.8	5.8	6.2
Med	7.1	7.4	5.8	7.3	5.7	5.3	5.1	5.1	6.1	6.0	6.1	6.5
75%	7.5	8.0	6.1	7.7	5.9	5.5	5.3	5.2	6.4	6.1	6.4	6.7
Max	9.2	9.8	7.1	9.8	6.5	6.3	5.9	5.7	7.1	6.7	7.3	7.9

Skew coefficient (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.81	4.60	3.41	4.03	2.18	2.99	2.38	1.79	2.57	2.15	3.18	2.31
Gen mean	3.50	3.98	3.28	3.90	2.19	3.03	2.40	1.84	2.63	2.18	3.10	2.33
StDev	0.88	0.89	0.65	0.79	0.28	0.52	0.30	0.26	0.35	0.23	0.71	0.26
Min	2.12	2.29	2.08	2.41	1.53	2.00	1.81	1.27	1.71	1.72	1.97	1.90
25%	3.02	3.36	2.87	3.44	2.00	2.63	2.19	1.67	2.38	2.07	2.66	2.16
Med	3.32	3.82	3.21	3.71	2.16	2.95	2.40	1.81	2.65	2.15	2.88	2.32
75%	3.84	4.23	3.60	4.20	2.34	3.35	2.56	2.02	2.78	2.29	3.40	2.44
Max	8.51	7.27	6.22	7.72	3.16	4.64	3.36	2.96	4.03	3.22	5.68	3.54

Wet 1 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.55	1.30	1.44	0.94	0.88	0.50	0.40	0.59	0.65	2.01	1.05	0.92
Gen mean	1.83	1.40	1.51	1.14	1.14	1.00	1.11	1.06	1.27	1.66	1.47	1.62
StDev	0.42	0.40	0.34	0.27	0.35	0.32	0.48	0.35	0.36	0.44	0.39	0.39
Min	0.89	0.71	0.85	0.57	0.49	0.45	0.47	0.48	0.69	0.79	0.76	0.84
25%	1.52	1.12	1.25	0.94	0.87	0.77	0.76	0.86	0.99	1.34	1.23	1.37
Med	1.79	1.31	1.49	1.14	1.10	0.97	1.02	0.97	1.21	1.57	1.41	1.57
75%	2.00	1.64	1.68	1.36	1.37	1.18	1.31	1.23	1.51	1.82	1.70	1.81
Max	3.15	3.16	2.47	1.86	1.98	2.07	2.77	2.41	2.38	3.00	2.89	2.86

Wet 2 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.05	2.48	2.37	2.31	2.32	1.81	1.97	2.04	2.47	2.59	2.57	2.89
Gen mean	3.47	3.24	3.10	2.86	2.87	2.30	2.60	2.65	3.00	3.38	3.19	3.43
StDev	0.41	0.48	0.31	0.37	0.25	0.30	0.25	0.28	0.28	0.30	0.31	0.33
Min	2.29	2.29	2.26	2.15	2.12	1.77	1.89	2.15	2.42	2.35	2.58	2.65
25%	3.18	2.88	2.89	2.59	2.71	2.10	2.41	2.47	2.82	3.16	2.99	3.20
Med	3.44	3.17	3.08	2.78	2.87	2.25	2.60	2.63	2.96	3.37	3.16	3.42
75%	3.77	3.57	3.29	3.15	3.03	2.46	2.77	2.83	3.15	3.58	3.39	3.65
Max	4.39	4.53	3.91	3.64	3.55	3.20	3.33	3.35	3.76	4.46	4.18	4.20

Table A.2 : Statistics for Daily Rainfall using Transition Probability Matrix Method (continued)

Wet 3 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	5.67	6.03	5.14	5.58	5.24	4.21	4.37	5.23	5.61	5.56	5.36	6.16
Gen mean	5.91	5.89	5.01	5.81	5.35	4.52	4.63	5.44	5.85	5.76	5.47	6.17
StDev	0.63	0.67	0.40	0.49	0.31	0.27	0.22	0.23	0.31	0.28	0.34	0.49
Min	4.56	4.11	4.29	4.88	4.41	3.74	4.07	4.97	5.09	5.21	4.59	4.92
25%	5.49	5.51	4.69	5.48	5.16	4.31	4.49	5.27	5.63	5.55	5.26	5.85
Med	5.91	5.82	5.02	5.73	5.40	4.51	4.64	5.45	5.81	5.76	5.44	6.14
75%	6.27	6.36	5.30	6.14	5.56	4.70	4.78	5.59	6.06	5.95	5.69	6.54
Max	7.59	8.44	6.06	7.01	5.85	5.34	5.21	5.97	6.69	6.54	6.42	7.59

Dry spell mean (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.3	3.4	2.8	3.0	2.0	1.8	1.6	2.0	2.1	2.3	2.3	3.1
Gen mean	3.4	3.5	2.9	2.9	2.1	1.8	1.6	2.0	2.1	2.3	2.4	3.1
StDev	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
Min	3.0	3.0	2.5	2.5	1.7	1.6	1.3	1.8	1.8	2.1	2.1	2.4
25%	3.3	3.3	2.8	2.8	2.0	1.8	1.5	1.9	2.0	2.3	2.3	3.0
Med	3.4	3.5	2.9	2.8	2.1	1.8	1.6	2.0	2.1	2.3	2.4	3.1
75%	3.5	3.6	3.0	3.0	2.1	1.9	1.6	2.1	2.1	2.4	2.5	3.1
Max	3.9	4.1	3.4	3.4	2.4	2.0	1.9	2.2	2.4	2.7	2.9	3.5

Dry spell standard deviation (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.6	2.6	2.2	2.4	1.4	1.0	0.8	1.5	1.3	1.7	1.7	2.6
Gen mean	2.9	2.9	2.4	2.3	1.5	1.2	1.0	1.4	1.5	1.8	1.9	2.5
StDev	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2
Min	2.4	2.3	1.9	1.8	1.1	0.9	0.7	1.0	1.2	1.4	1.6	2.0
25%	2.8	2.7	2.2	2.1	1.3	1.1	0.9	1.3	1.4	1.7	1.7	2.4
Med	2.9	2.9	2.3	2.2	1.5	1.2	1.0	1.4	1.5	1.8	1.8	2.5
75%	3.1	3.1	2.5	2.4	1.6	1.3	1.1	1.6	1.6	1.9	2.0	2.7
Max	4.5	3.7	3.0	3.0	1.8	1.7	1.4	2.0	2.0	2.3	2.4	3.1

Dry spell skew (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.70	1.74	1.83	1.69	1.82	1.31	1.26	1.63	1.80	2.04	1.35	1.76
Gen mean	2.07	1.88	1.94	1.83	1.97	2.04	2.16	1.97	2.14	1.92	2.07	1.95
StDev	0.67	0.42	0.46	0.46	0.45	0.49	0.55	0.48	0.60	0.37	0.48	0.41
Min	1.16	1.05	0.96	1.09	1.22	1.15	1.34	1.28	0.93	1.23	1.35	1.15
25%	1.68	1.59	1.63	1.51	1.64	1.76	1.81	1.62	1.74	1.67	1.73	1.69
Med	1.86	1.85	1.83	1.71	1.92	1.98	2.05	1.85	2.05	1.89	1.94	1.86
75%	2.25	2.10	2.17	2.05	2.25	2.20	2.43	2.19	2.35	2.12	2.23	2.18
Max	5.09	3.35	3.44	3.20	3.78	4.28	5.37	3.78	5.09	3.35	3.73	3.11

Wet spell mean (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.7	2.7	3.3	4.4	6.1	9.1	7.8	8.1	6.2	4.6	4.0	3.7
Gen mean	2.7	2.7	3.1	4.3	6.0	7.1	7.5	7.0	5.4	4.5	4.0	3.3
StDev	0.1	0.2	0.2	0.3	0.4	0.6	0.6	0.6	0.4	0.3	0.2	0.2
Min	2.5	2.3	2.7	3.6	4.9	5.7	6.1	5.7	4.7	3.9	3.6	2.7
25%	2.6	2.6	3.0	4.0	5.6	6.7	7.1	6.6	5.1	4.3	3.8	3.1
Med	2.7	2.7	3.1	4.2	6.0	7.0	7.5	7.0	5.4	4.5	4.0	3.3
75%	2.8	2.8	3.3	4.5	6.3	7.5	7.9	7.4	5.6	4.7	4.2	3.4
Max	3.2	3.2	3.8	5.2	6.9	8.6	8.9	8.6	6.6	5.4	4.6	3.8

Table A.2 : Statistics for Daily Rainfall using Transition Probability Matrix Method (continued)

Wet spell standard deviation (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.1	2.0	3.1	4.1	5.1	8.6	6.4	6.7	5.6	4.3	3.1	2.9
Gen mean	2.2	2.2	2.8	4.1	6.0	7.0	7.2	6.6	4.9	4.1	3.5	2.8
StDev	0.2	0.2	0.3	0.5	0.8	0.9	0.8	0.7	0.5	0.4	0.3	0.3
Min	1.7	1.7	2.0	3.0	4.5	5.2	5.7	5.2	3.7	3.3	2.9	2.1
25%	2.1	2.1	2.6	3.8	5.4	6.5	6.7	6.1	4.6	3.8	3.3	2.6
Med	2.2	2.2	2.7	4.1	5.9	6.9	7.2	6.5	4.8	4.0	3.5	2.7
75%	2.4	2.4	2.9	4.5	6.5	7.6	7.7	7.0	5.2	4.3	3.7	2.9
Max	2.9	2.8	3.6	5.4	8.0	9.2	9.5	8.0	6.5	5.1	4.6	3.7

Wet spell skew (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.31	1.79	3.08	1.75	2.08	1.66	1.56	1.11	1.49	2.06	1.53	2.10
Gen mean	2.13	2.01	2.20	2.21	2.09	2.03	1.82	1.67	1.76	1.86	1.87	1.89
StDev	0.62	0.45	0.61	0.47	0.55	0.59	0.42	0.40	0.40	0.40	0.39	0.37
Min	1.15	1.20	1.23	1.20	1.18	1.08	0.94	0.97	0.98	1.08	1.12	1.18
25%	1.75	1.70	1.74	1.88	1.65	1.62	1.56	1.38	1.49	1.60	1.59	1.64
Med	2.00	1.89	2.07	2.14	2.02	1.99	1.77	1.62	1.67	1.76	1.81	1.82
75%	2.32	2.20	2.47	2.49	2.44	2.22	2.01	1.84	1.95	2.18	2.06	2.04
Max	4.61	3.55	4.63	3.38	4.24	4.89	3.06	2.85	3.32	2.79	3.41	3.03

Number of wet days (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	14.3	12.7	16.3	17.9	23.1	24.4	26.3	25.0	22.5	21.1	19.6	16.8
Gen mean	13.8	12.2	15.9	17.2	22.7	23.7	25.7	24.5	22.1	20.5	19.1	16.3
StDev	0.6	0.6	0.6	0.6	0.6	0.5	0.4	0.6	0.5	0.6	0.6	0.6
Min	12.2	10.4	14.5	15.9	21.0	22.6	24.8	23.0	21.1	19.1	17.5	14.9
25%	13.4	11.8	15.4	16.8	22.3	23.4	25.3	24.1	21.7	20.2	18.6	15.9
Med	13.7	12.2	15.9	17.2	22.8	23.7	25.7	24.5	22.1	20.6	19.1	16.3
75%	14.1	12.5	16.3	17.6	23.1	24.0	26.0	24.9	22.5	20.9	19.5	16.7
Max	15.2	13.7	17.1	18.9	24.1	25.1	26.5	26.0	23.2	21.7	20.3	18.9

APPENDIX B

MODIFIED WANG-NATHAN MODEL RESULTS

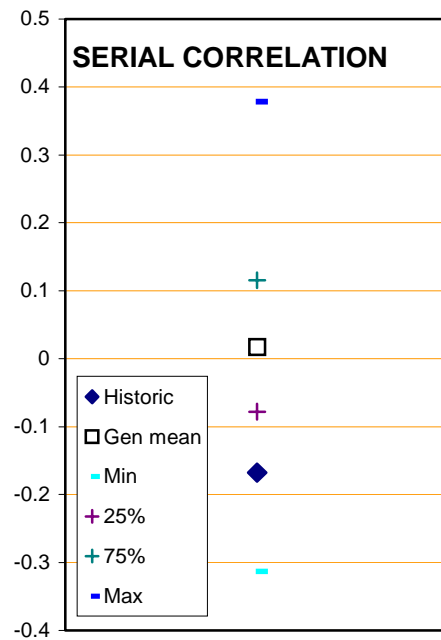
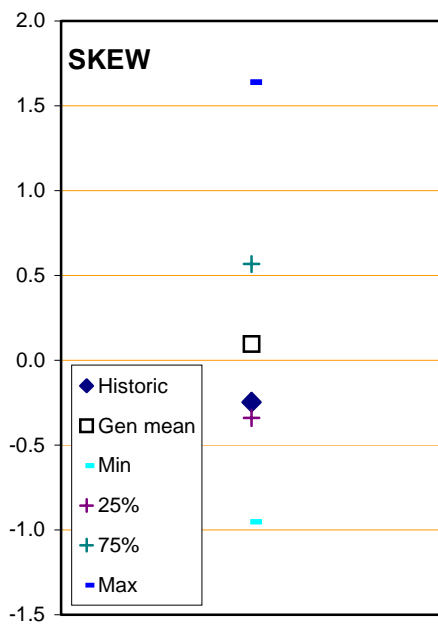
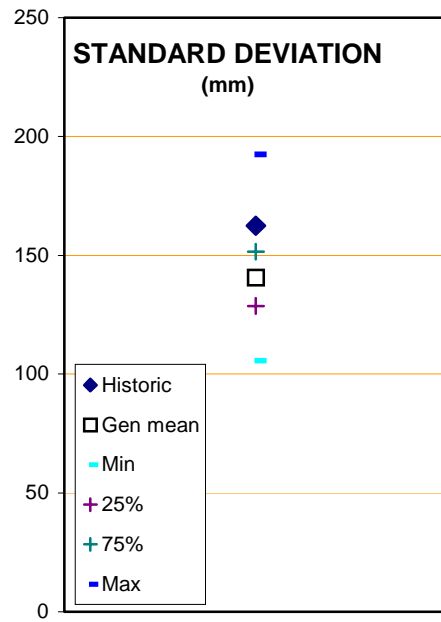
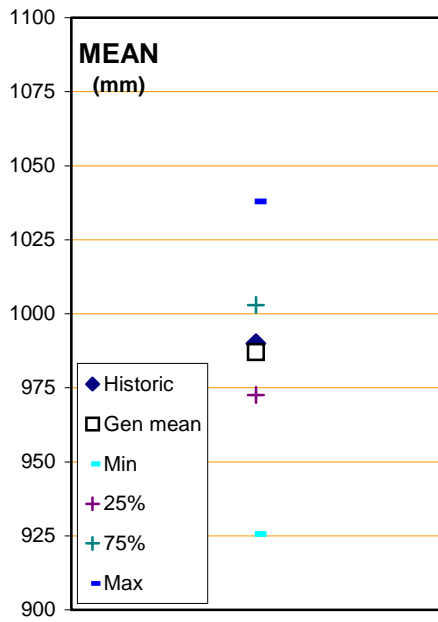


Figure B.1: Evaluation of annual rainfall statistics for modified Wang_Nathan Model (mean, standard deviation, skew and serial correlation)

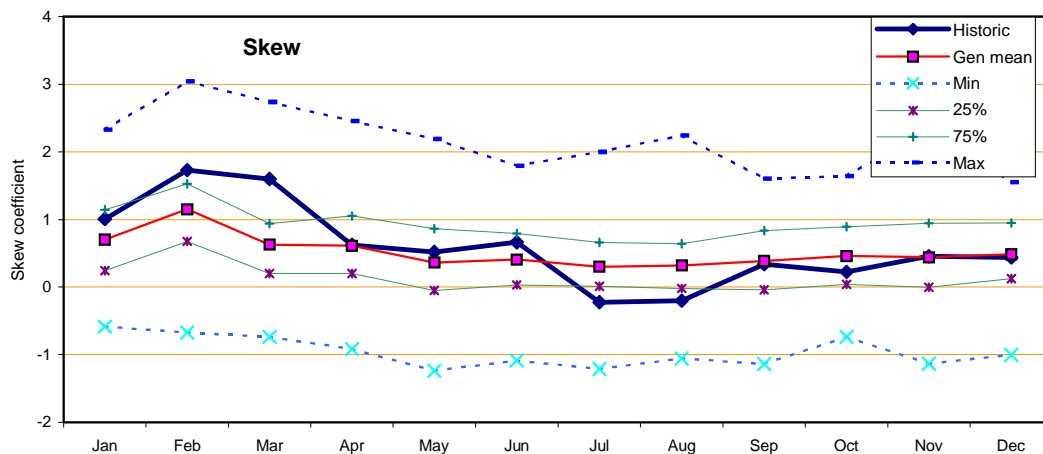
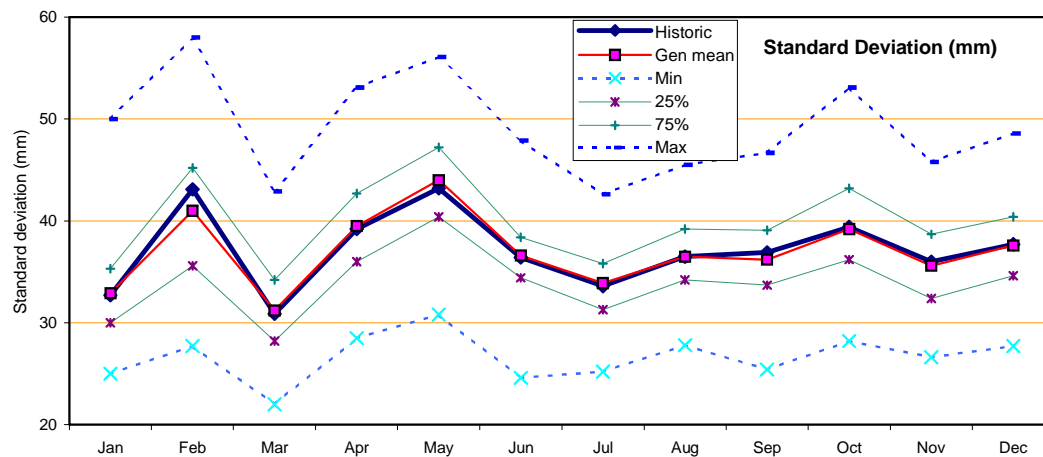
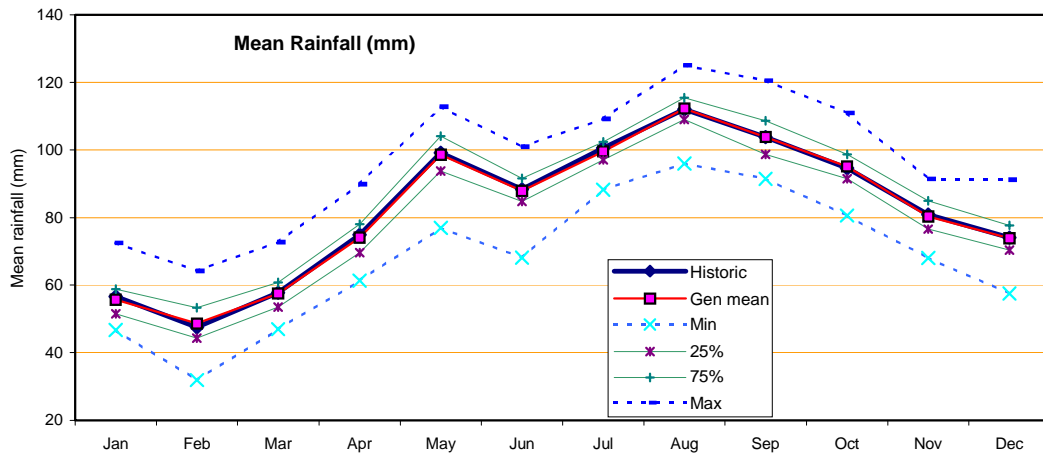


Figure B.2(a) : Evaluation of monthly rainfall statistics (mean, standard deviation, skew) modified Wang-Nathan Model (WNM)

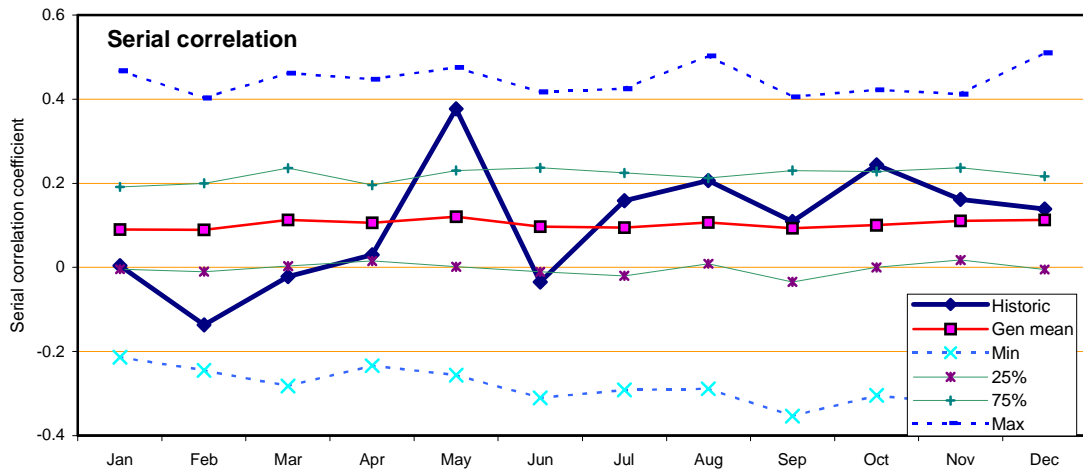


Figure B.2(b) : Evaluation of monthly rainfall statistics (serial correlation) modified Wang-Nathan Model (WNM)

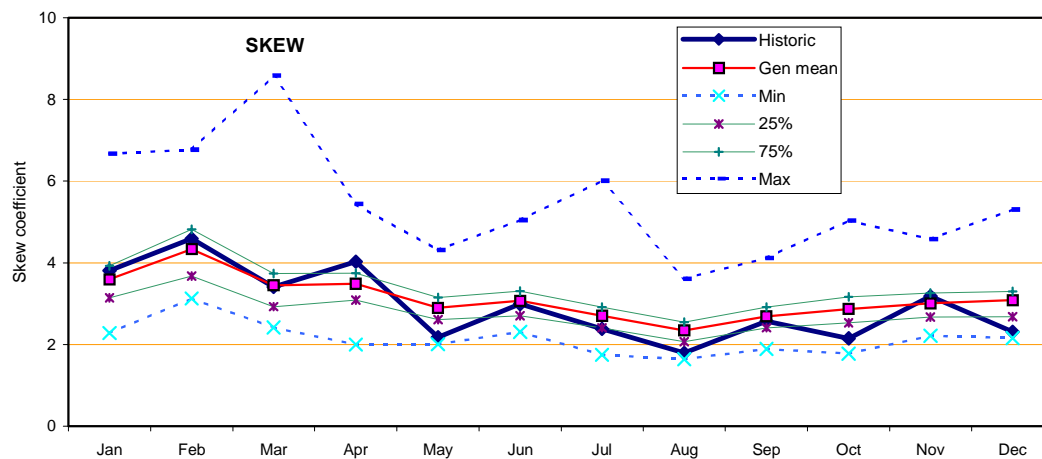
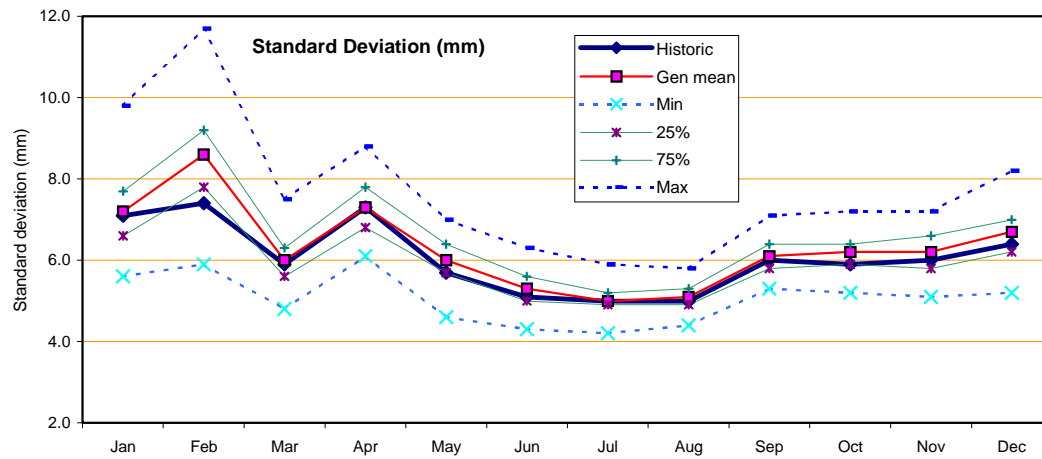
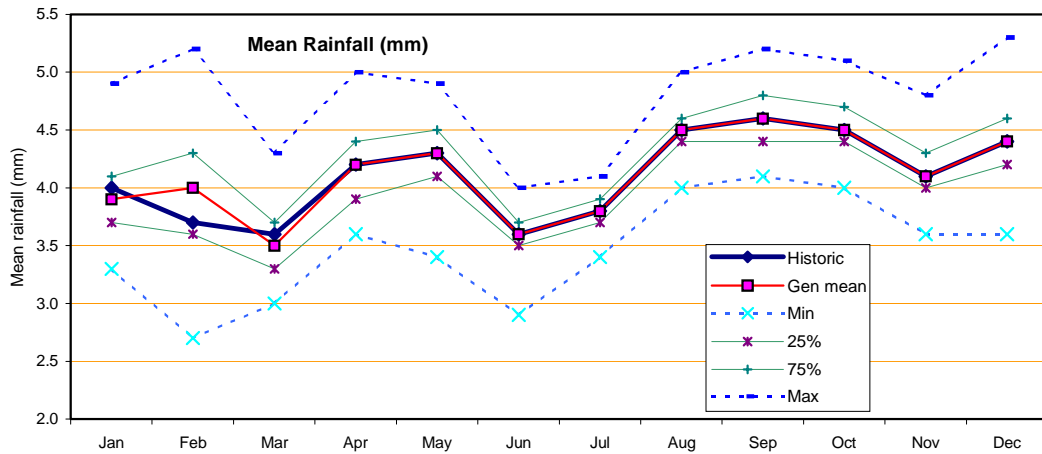


Figure B.3(a): Evaluation of daily rainfall statistics (mean, standard deviation, skew) modified Wang-Nathan Model (WNM)

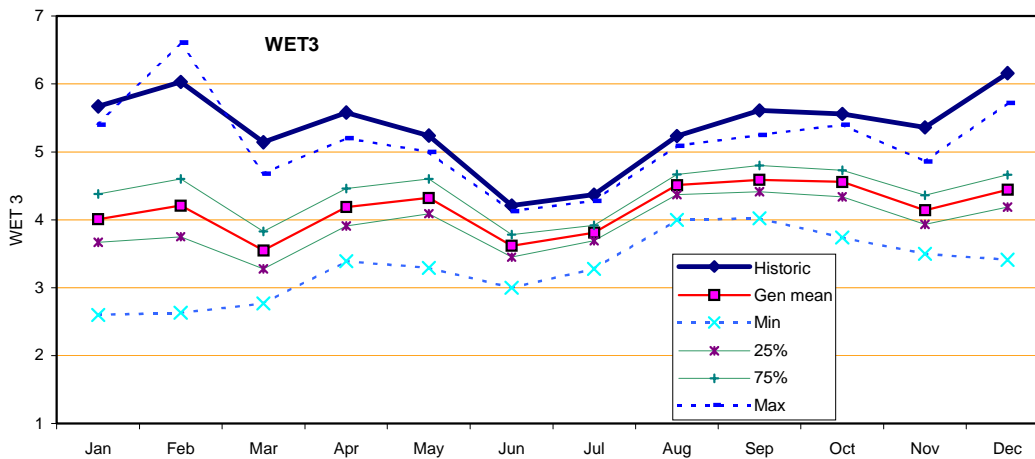
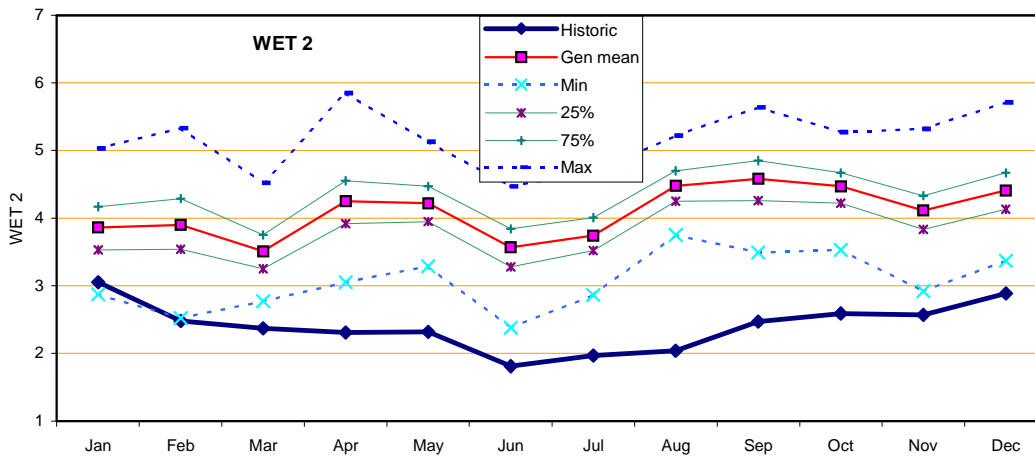
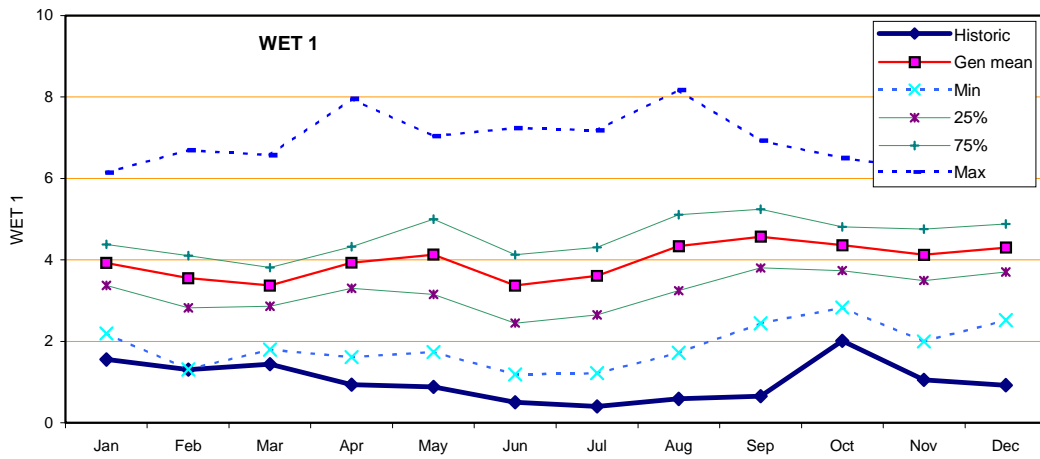


Figure B.3(b): Evaluation of daily rainfall statistics (WET 1,WET 2,WET 3) modified Wang-Nathan Model (WNM)

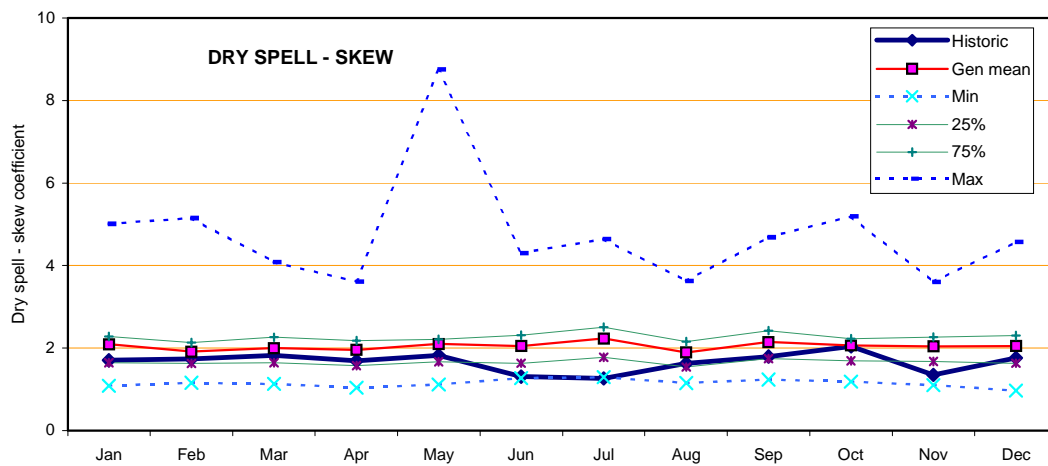
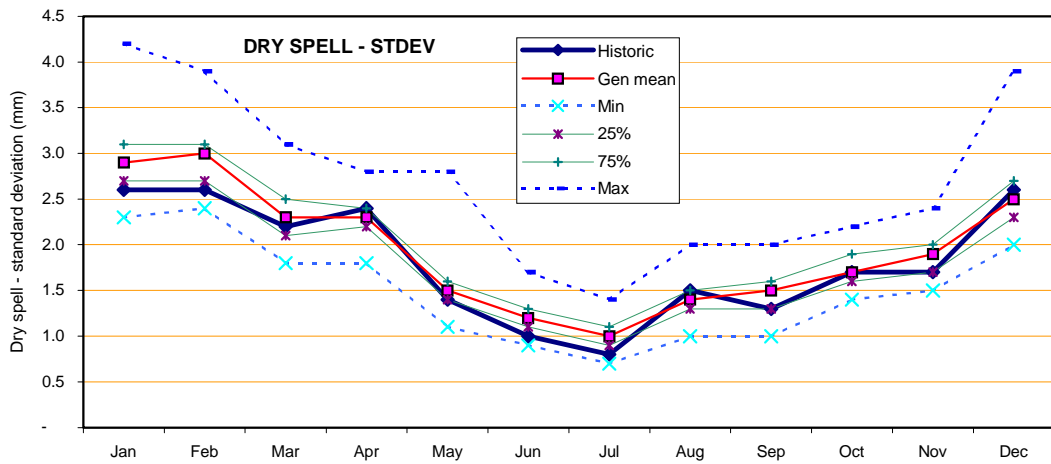
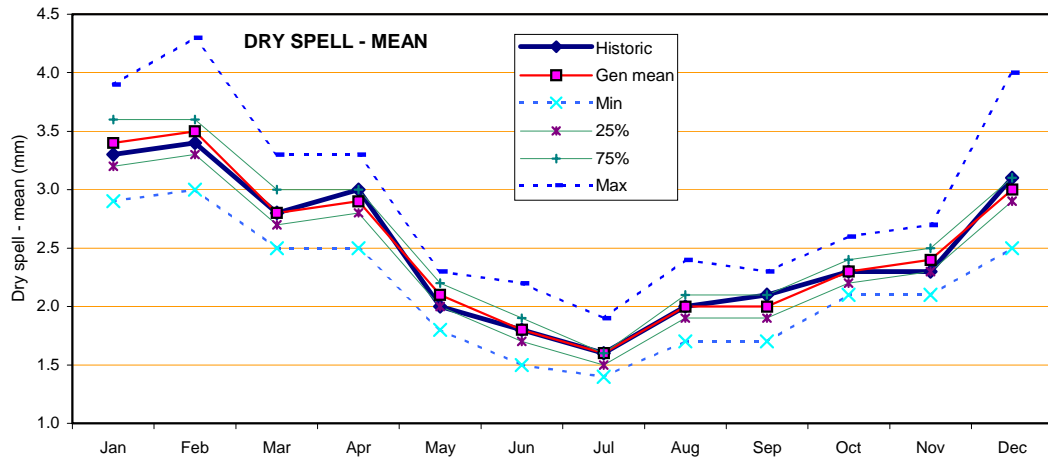


Figure B.3(c): Evaluation of daily rainfall statistics (dry spell mean, standard deviation, skew) modified Wang-Nathan Model (WNM)

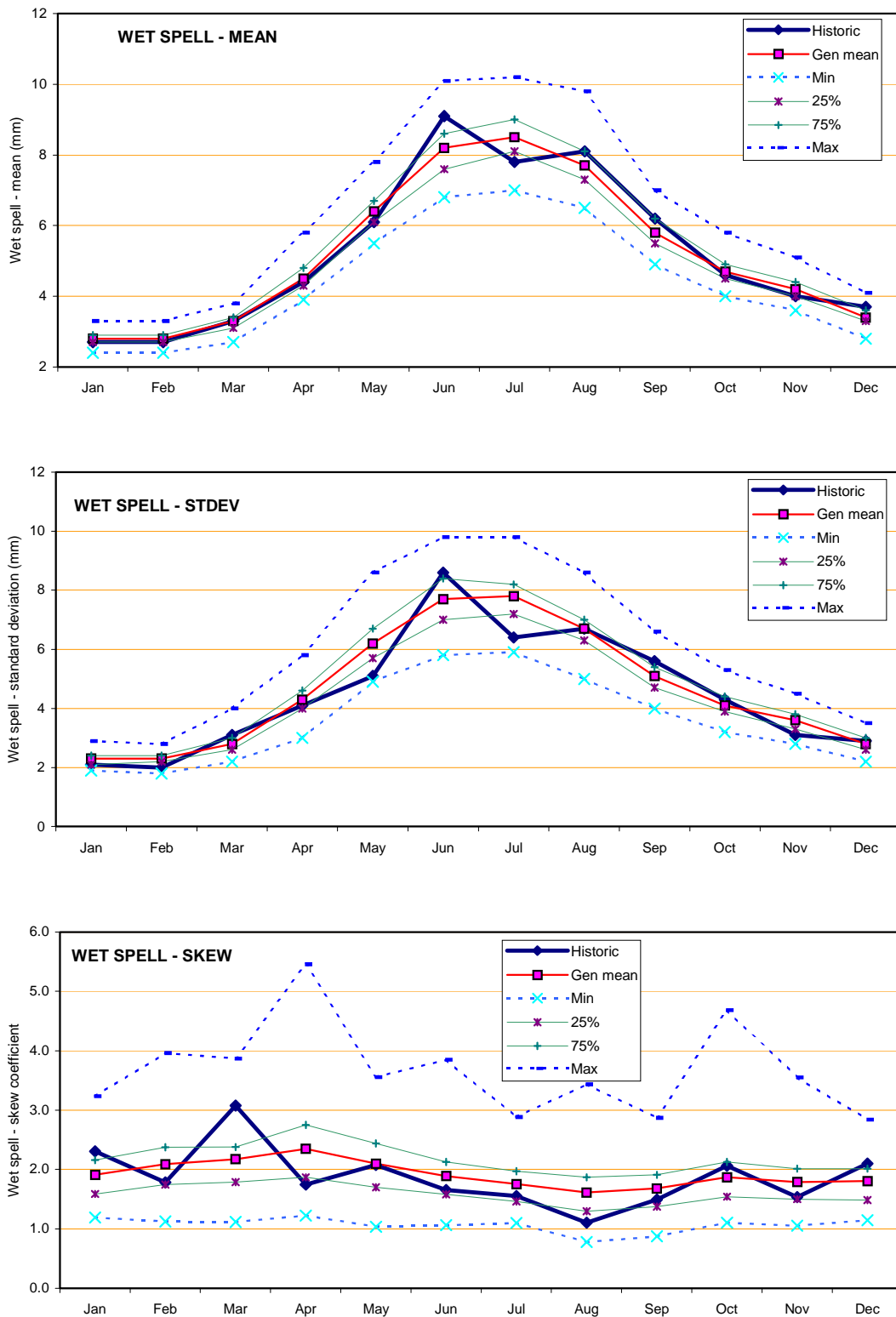


Figure B.3(d): Evaluation of daily rainfall statistics (wet spell mean, standard deviation, skew) modified Wang-Nathan Model (WNM)

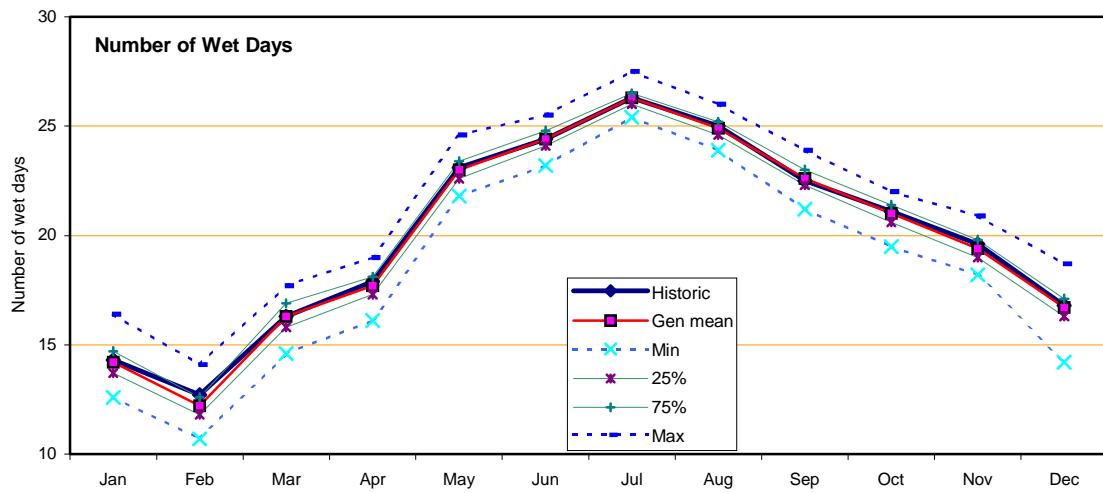


Figure B.3(e): Evaluation of daily rainfall statistics (number of wet days) modified Wang-Nathan Model (WNM)

Table B.1: Annual and monthly statistics for generated rainfall using modified Wang-Nathan Model

ANNUAL STATISTICS

	Annual	StDev	Skew	R1	Min	Max	Range	2-year	3-year	5-year	7-year	10-year
Historic	990	162	-0.25	-0.17	0.64	1.33	1.00	1.62	2.55	4.37	6.24	9.19
Gen mean	987	141	0.10	0.02	0.71	1.31	1.28	1.58	2.50	4.40	6.33	9.30
StDev	22	16	0.53	0.14	0.05	0.07	0.42	0.09	0.12	0.17	0.24	0.30
Min	926	106	-0.95	-0.31	0.56	1.17	0.51	1.26	2.16	3.95	5.77	8.56
25%	973	129	-0.34	-0.08	0.68	1.25	1.03	1.52	2.42	4.27	6.18	9.10
Med	987	141	0.09	0.00	0.72	1.30	1.20	1.58	2.49	4.42	6.39	9.31
75%	1003	152	0.57	0.12	0.75	1.36	1.53	1.64	2.58	4.54	6.51	9.52
Max	1038	192	1.64	0.38	0.88	1.58	2.84	1.80	2.82	4.81	6.77	9.88

MONTHLY STATISTICS

Mean monthly rainfall (mm)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	56.6	47.5	57.8	74.9	99.1	88.3	100.6	112.0	103.8	94.6	80.8	74.0
Gen mean	55.7	48.6	57.5	74.0	98.6	87.9	99.5	112.2	103.8	95.1	80.3	73.9
StDev	5.3	6.4	5.3	6.1	7.7	5.8	4.6	5.5	6.7	6.0	5.4	6.1
Min	46.7	31.9	47.0	61.3	76.9	68.1	88.2	96.0	91.4	80.6	68.0	57.5
25%	51.5	44.3	53.5	69.6	93.7	84.7	97.0	109.0	98.7	91.4	76.6	70.4
Med	55.7	49.0	57.1	74.5	99.6	88.2	99.4	112.8	103.3	95.3	80.3	73.8
75%	58.8	53.3	60.8	78.0	104.1	91.5	102.4	115.5	108.6	98.7	84.9	77.7
Max	72.5	64.2	72.7	89.9	112.8	100.9	109.2	125.0	120.5	110.9	91.4	91.2

Standard Deviation (monthly rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	32.7	43.1	30.9	39.2	43.2	36.4	33.6	36.5	36.9	39.4	36.0	37.7
Gen mean	32.9	41.0	31.2	39.5	44.0	36.6	33.9	36.5	36.2	39.2	35.6	37.6
StDev	4.5	6.1	4.5	5.1	5.1	4.1	3.6	3.6	4.1	5.4	4.4	4.5
Min	25.0	27.7	22.0	28.5	30.8	24.6	25.2	27.8	25.4	28.2	26.6	27.7
25%	30.0	35.6	28.2	36.0	40.4	34.4	31.3	34.2	33.7	36.2	32.4	34.6
Med	31.6	40.9	30.9	38.5	43.5	36.2	33.8	36.3	35.6	38.8	35.5	37.6
75%	35.3	45.2	34.2	42.7	47.2	38.4	35.8	39.2	39.1	43.2	38.7	40.4
Max	50.0	58.0	42.9	53.1	56.1	47.9	42.6	45.5	46.7	53.1	45.8	48.6

Skew coefficient (monthly rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.01	1.73	1.60	0.62	0.52	0.66	-0.23	-0.21	0.34	0.22	0.46	0.44
Gen mean	0.70	1.15	0.63	0.61	0.36	0.41	0.30	0.32	0.38	0.46	0.44	0.48
StDev	0.67	0.63	0.66	0.67	0.65	0.57	0.65	0.58	0.61	0.58	0.64	0.60
Min	-0.59	-0.67	-0.74	-0.92	-1.24	-1.09	-1.21	-1.06	-1.14	-0.74	-1.14	-1.01
25%	0.24	0.68	0.20	0.20	-0.05	0.03	0.01	-0.02	-0.04	0.04	0.00	0.12
Med	0.71	1.17	0.67	0.64	0.37	0.43	0.24	0.40	0.37	0.46	0.45	0.54
75%	1.14	1.53	0.94	1.05	0.86	0.79	0.66	0.64	0.83	0.89	0.94	0.95
Max	2.33	3.04	2.74	2.46	2.19	1.80	2.00	2.24	1.60	1.64	2.32	1.55

Serial correlation (monthly rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	0.00	-0.14	-0.02	0.03	0.38	-0.03	0.16	0.21	0.11	0.24	0.16	0.14
Gen mean	0.09	0.09	0.11	0.11	0.12	0.10	0.09	0.11	0.09	0.10	0.11	0.11
StDev	0.14	0.14	0.15	0.14	0.16	0.15	0.16	0.15	0.17	0.16	0.15	0.15
Min	-0.21	-0.24	-0.28	-0.23	-0.26	-0.31	-0.29	-0.29	-0.35	-0.30	-0.33	-0.23
25%	0.00	-0.01	0.00	0.02	0.00	-0.01	-0.02	0.01	-0.03	0.00	0.02	-0.01
Med	0.06	0.10	0.12	0.11	0.12	0.09	0.09	0.11	0.11	0.10	0.10	0.12
75%	0.19	0.20	0.24	0.20	0.23	0.24	0.22	0.21	0.23	0.23	0.24	0.22
Max	0.47	0.40	0.46	0.45	0.48	0.42	0.43	0.50	0.41	0.42	0.41	0.51

Table B.2 : Statistics for Daily Rainfall using modified Wang-Nathan Model (WNM)

Mean daily rainfall (mm)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	4.0	3.7	3.6	4.2	4.3	3.6	3.8	4.5	4.6	4.5	4.1	4.4
Gen mean	3.9	4.0	3.5	4.2	4.3	3.6	3.8	4.5	4.6	4.5	4.1	4.4
StDev	0.3	0.4	0.3	0.3	0.3	0.2	0.2	0.2	0.3	0.2	0.3	0.3
Min	3.3	2.7	3.0	3.6	3.4	2.9	3.4	4.0	4.1	4.0	3.6	3.6
25%	3.7	3.6	3.3	3.9	4.1	3.5	3.7	4.4	4.4	4.4	4.0	4.2
Med	3.9	4.0	3.5	4.2	4.3	3.6	3.8	4.5	4.6	4.5	4.1	4.4
75%	4.1	4.3	3.7	4.4	4.5	3.7	3.9	4.6	4.8	4.7	4.3	4.6
Max	4.9	5.2	4.3	5.0	4.9	4.0	4.1	5.0	5.2	5.1	4.8	5.3

Standard Deviation (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	7.1	7.4	5.9	7.3	5.7	5.1	5.0	5.0	6.0	5.9	6.0	6.4
Gen mean	7.2	8.6	6.0	7.3	6.0	5.3	5.0	5.1	6.1	6.2	6.2	6.7
StDev	0.8	1.1	0.6	0.7	0.5	0.4	0.3	0.3	0.4	0.5	0.5	0.6
Min	5.6	5.9	4.8	6.1	4.6	4.3	4.2	4.4	5.3	5.2	5.1	5.2
25%	6.6	7.8	5.6	6.8	5.7	5.0	4.9	4.9	5.8	5.9	5.8	6.2
Med	7.2	8.5	5.9	7.3	6.0	5.3	5.0	5.1	6.1	6.2	6.1	6.7
75%	7.7	9.2	6.3	7.8	6.4	5.6	5.2	5.3	6.4	6.4	6.6	7.0
Max	9.8	11.7	7.5	8.8	7.0	6.3	5.9	5.8	7.1	7.2	7.2	8.2

Skew coefficient (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.81	4.60	3.41	4.03	2.18	2.99	2.38	1.79	2.57	2.15	3.18	2.31
Gen mean	3.59	4.34	3.45	3.49	2.90	3.07	2.70	2.35	2.69	2.87	3.00	3.09
StDev	0.71	0.81	0.89	0.61	0.41	0.49	0.52	0.37	0.42	0.53	0.48	0.57
Min	2.28	3.13	2.42	2.00	2.00	2.31	1.74	1.64	1.89	1.78	2.21	2.15
25%	3.14	3.68	2.92	3.09	2.61	2.70	2.41	2.07	2.41	2.53	2.67	2.68
Med	3.43	4.28	3.27	3.36	2.88	2.99	2.61	2.34	2.62	2.85	2.97	2.98
75%	3.93	4.81	3.74	3.75	3.15	3.31	2.91	2.55	2.91	3.16	3.26	3.30
Max	6.68	6.77	8.58	5.45	4.31	5.05	6.01	3.61	4.12	5.03	4.58	5.31

Wet 1 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.55	1.30	1.44	0.94	0.88	0.50	0.40	0.59	0.65	2.01	1.05	0.92
Gen mean	3.92	3.55	3.37	3.93	4.13	3.37	3.61	4.34	4.57	4.36	4.12	4.30
StDev	0.85	1.05	0.80	1.12	1.20	1.22	1.29	1.40	1.04	0.83	0.93	0.96
Min	2.19	1.31	1.80	1.61	1.74	1.19	1.22	1.72	2.44	2.83	2.00	2.52
25%	3.37	2.82	2.86	3.30	3.15	2.45	2.65	3.24	3.80	3.74	3.49	3.70
Med	3.80	3.37	3.40	3.72	4.16	3.17	3.50	4.18	4.60	4.29	3.97	4.19
75%	4.38	4.10	3.81	4.32	5.00	4.12	4.31	5.11	5.24	4.81	4.75	4.88
Max	6.14	6.69	6.57	7.95	7.04	7.24	7.18	8.17	6.93	6.50	6.24	6.92

Wet 2 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.05	2.48	2.37	2.31	2.32	1.81	1.97	2.04	2.47	2.59	2.57	2.89
Gen mean	3.86	3.90	3.51	4.25	4.22	3.57	3.74	4.48	4.58	4.47	4.11	4.41
StDev	0.48	0.57	0.39	0.51	0.40	0.41	0.34	0.35	0.44	0.36	0.40	0.42
Min	2.87	2.52	2.77	3.05	3.29	2.38	2.86	3.75	3.49	3.53	2.92	3.37
25%	3.53	3.54	3.25	3.92	3.95	3.28	3.52	4.25	4.26	4.22	3.83	4.13
Med	3.81	3.94	3.46	4.23	4.20	3.57	3.74	4.48	4.59	4.47	4.05	4.40
75%	4.17	4.29	3.75	4.55	4.47	3.84	4.01	4.70	4.85	4.67	4.33	4.67
Max	5.03	5.33	4.52	5.85	5.13	4.47	4.68	5.22	5.64	5.27	5.32	5.71

Table B.2 : Statistics for Daily Rainfall using modified Wang-Nathan Model (continued)

Wet 3 (daily rainfall) - mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	5.67	6.03	5.14	5.58	5.24	4.21	4.37	5.23	5.61	5.56	5.36	6.16
Gen mean	4.01	4.21	3.55	4.19	4.32	3.62	3.81	4.51	4.59	4.56	4.14	4.44
StDev	0.53	0.67	0.36	0.37	0.35	0.24	0.19	0.21	0.27	0.32	0.30	0.39
Min	2.60	2.63	2.77	3.39	3.29	3.00	3.28	4.00	4.02	3.74	3.50	3.41
25%	3.67	3.75	3.28	3.91	4.09	3.45	3.69	4.37	4.41	4.34	3.93	4.19
Med	3.95	4.26	3.51	4.17	4.28	3.63	3.80	4.48	4.57	4.58	4.14	4.46
75%	4.38	4.60	3.83	4.46	4.60	3.78	3.92	4.67	4.80	4.73	4.36	4.66
Max	5.40	6.61	4.68	5.20	5.00	4.13	4.28	5.09	5.25	5.40	4.86	5.72

Dry spell mean (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	3.3	3.4	2.8	3.0	2.0	1.8	1.6	2.0	2.1	2.3	2.3	3.1
Gen mean	3.4	3.5	2.8	2.9	2.1	1.8	1.6	2.0	2.0	2.3	2.4	3.0
StDev	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
Min	2.9	3.0	2.5	2.5	1.8	1.5	1.4	1.7	1.7	2.1	2.1	2.5
25%	3.2	3.3	2.7	2.8	2.0	1.7	1.5	1.9	1.9	2.2	2.3	2.9
Med	3.4	3.5	2.8	2.9	2.1	1.8	1.6	2.0	2.0	2.3	2.4	3.0
75%	3.6	3.6	3.0	3.0	2.2	1.9	1.6	2.1	2.1	2.4	2.5	3.1
Max	3.9	4.3	3.3	3.3	2.3	2.2	1.9	2.4	2.3	2.6	2.7	4.0

Dry spell standard deviation (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.6	2.6	2.2	2.4	1.4	1.0	0.8	1.5	1.3	1.7	1.7	2.6
Gen mean	2.9	3.0	2.3	2.3	1.5	1.2	1.0	1.4	1.5	1.7	1.9	2.5
StDev	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.3
Min	2.3	2.4	1.8	1.8	1.1	0.9	0.7	1.0	1.0	1.4	1.5	2.0
25%	2.7	2.7	2.1	2.2	1.4	1.1	0.9	1.3	1.3	1.6	1.7	2.3
Med	2.9	3.0	2.3	2.3	1.5	1.2	0.9	1.4	1.5	1.7	1.9	2.5
75%	3.1	3.1	2.5	2.4	1.6	1.3	1.1	1.5	1.6	1.9	2.0	2.7
Max	4.2	3.9	3.1	2.8	2.8	1.7	1.4	2.0	2.0	2.2	2.4	3.9

Dry spell skew (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	1.70	1.74	1.83	1.69	1.82	1.31	1.26	1.63	1.80	2.04	1.35	1.76
Gen mean	2.09	1.92	2.00	1.96	2.10	2.05	2.23	1.89	2.15	2.07	2.04	2.05
StDev	0.73	0.51	0.45	0.51	1.02	0.55	0.66	0.45	0.59	0.62	0.46	0.58
Min	1.09	1.16	1.13	1.04	1.12	1.27	1.30	1.15	1.24	1.19	1.10	0.97
25%	1.64	1.62	1.64	1.58	1.67	1.63	1.77	1.54	1.74	1.69	1.67	1.64
Med	1.92	1.81	1.92	1.92	1.94	1.94	2.11	1.82	1.97	1.95	2.01	1.89
75%	2.29	2.13	2.26	2.19	2.21	2.31	2.51	2.16	2.43	2.22	2.27	2.30
Max	5.01	5.15	4.09	3.61	8.75	4.30	4.64	3.63	4.69	5.19	3.60	4.57

Wet spell mean (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.7	2.7	3.3	4.4	6.1	9.1	7.8	8.1	6.2	4.6	4.0	3.7
Gen mean	2.8	2.8	3.3	4.5	6.4	8.2	8.5	7.7	5.8	4.7	4.2	3.4
StDev	0.2	0.2	0.2	0.4	0.5	0.7	0.7	0.6	0.4	0.3	0.3	0.2
Min	2.4	2.4	2.7	3.9	5.5	6.8	7.0	6.5	4.9	4.0	3.6	2.8
25%	2.7	2.7	3.1	4.3	6.1	7.6	8.1	7.3	5.5	4.5	4.0	3.3
Med	2.8	2.8	3.2	4.5	6.4	8.1	8.4	7.6	5.8	4.7	4.2	3.4
75%	2.9	2.9	3.4	4.8	6.7	8.6	9.0	8.1	6.2	4.9	4.4	3.6
Max	3.3	3.3	3.8	5.8	7.8	10.1	10.2	9.8	7.0	5.8	5.1	4.1

Table B.2 : Statistics for Daily Rainfall using modified Wang-Nathan Model (continued)

Wet spell standard deviation (daily rainfall) - days

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.1	2.0	3.1	4.1	5.1	8.6	6.4	6.7	5.6	4.3	3.1	2.9
Gen mean	2.3	2.3	2.8	4.3	6.2	7.7	7.8	6.7	5.1	4.1	3.6	2.8
StDev	0.2	0.2	0.3	0.6	0.7	0.9	0.8	0.7	0.5	0.4	0.4	0.3
Min	1.9	1.8	2.2	3.0	4.9	5.8	5.9	5.0	4.0	3.2	2.8	2.2
25%	2.1	2.2	2.6	4.0	5.7	7.0	7.2	6.3	4.7	3.9	3.3	2.6
Med	2.3	2.3	2.8	4.2	6.1	7.6	7.7	6.7	5.1	4.0	3.6	2.8
75%	2.4	2.4	3.0	4.6	6.7	8.4	8.2	7.0	5.4	4.4	3.8	3.0
Max	2.9	2.8	4.0	5.8	8.6	9.8	9.8	8.6	6.6	5.3	4.5	3.5

Wet spell skew (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	2.31	1.79	3.08	1.75	2.08	1.66	1.56	1.11	1.49	2.06	1.53	2.10
Gen mean	1.91	2.09	2.17	2.35	2.10	1.89	1.76	1.61	1.68	1.87	1.79	1.81
StDev	0.43	0.53	0.60	0.70	0.54	0.45	0.38	0.45	0.40	0.48	0.41	0.41
Min	1.19	1.12	1.12	1.22	1.04	1.06	1.10	0.78	0.87	1.10	1.06	1.14
25%	1.59	1.75	1.79	1.87	1.70	1.58	1.46	1.29	1.37	1.54	1.50	1.48
Med	1.83	1.97	2.03	2.19	2.02	1.84	1.74	1.54	1.68	1.79	1.73	1.78
75%	2.16	2.37	2.38	2.75	2.44	2.13	1.97	1.87	1.91	2.13	2.02	2.01
Max	3.23	3.96	3.87	5.46	3.56	3.85	2.88	3.44	2.87	4.69	3.55	2.84

Number of wet days (daily rainfall)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historic	14.3	12.7	16.3	17.9	23.1	24.4	26.3	25.0	22.5	21.1	19.6	16.8
Gen mean	14.2	12.2	16.3	17.7	23.0	24.4	26.3	24.9	22.6	21.0	19.4	16.7
StDev	0.7	0.6	0.7	0.6	0.6	0.5	0.4	0.5	0.5	0.6	0.6	0.7
Min	12.6	10.7	14.6	16.1	21.8	23.2	25.4	23.9	21.2	19.5	18.2	14.2
25%	13.7	11.8	15.8	17.3	22.6	24.1	26.0	24.6	22.3	20.6	19.0	16.3
Med	14.2	12.2	16.4	17.7	23.0	24.4	26.3	24.9	22.6	21.1	19.4	16.7
75%	14.7	12.6	16.9	18.1	23.4	24.8	26.5	25.2	23.0	21.4	19.8	17.1
Max	16.4	14.1	17.7	19.0	24.6	25.5	27.5	26.0	23.9	22.0	20.9	18.7

