A REVIEW OF SCALE IN HYDROLOGY

A report as part of Project D2: Regionalisation and scaling of hydrologic data

G.C. Lacey

Report 95/1
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COOPERATIVE RESEARCH CENTRE FOR
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PREFACE

The Cooperative Research Centre (CRC) for Catchment Hydrology's research program "Flood Hydrology" has the overall objective:

> *To improve methods for estimating flood risk and the reliability of flood forecasting, and advance the understanding of catchment similarity and regional behaviour.*

The issues of catchment similarity and regional behaviour are specifically dealt with in CRC Project D2 'Regionalisation and Scaling of Hydrologic Data'. A report *Regionalisation of Hydrologic Data: A Review* (Bates 1994) has already been published as a CRC Report 94/5.

This report by Geoff Lacey (University of Melbourne) provides a review of the literature relating to scaling. This topic is relevant to many projects in the CRC; the extrapolation of results from test plots, or small research catchments, to much larger areas is a problem not limited to flood hydrology. Here, it is of interest to note that Geoff proposes to develop scaling theory suitable for the issue of catchment yield as an initial application.

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ABSTRACT

There are two aspects to the problem of scale in hydrology. The first is the recognition that different laws may dominate at different scales. The second involves the establishment of dimensionless parameters for a problem, so that a solution can be applied to a variety of actual systems. This paper examines the theory of scaling and the work done on the scaling of several hydrologic phenomena. The emphasis is on work involving real catchment data; however some modelling is also examined.

There has been reasonable success in scaling soil-water phenomena in the field. However, as yet there have been no comparable solutions involving data for larger scale phenomena. Work reviewed includes the scaling of surface saturation zones; the scaling of runoff generating processes and flood frequency using computer modelling; simple scaling and multiscaling theories and their application to flood peaks; and fractal theory. Recommendations are put forward for further research on the problem of scale.

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1. INTRODUCTION

1.1 The problem of scale

The importance of the problem of scale in hydrology is highlighted by Dooge (1986) in the following terms: 'To predict catchment behaviour reliably we must either solve extremely complex physically based models which take full account of the spatial variability of various parameters or else derive realistic models on the catchment scale in which the global effect of these spatially variable properties is parameterized in some way. The former approach requires extremely sophisticated models ... to have any hope of success. The latter approach requires the discovery of hydrologic laws at the catchment scale that represent more than mere data fitting. This is, indeed, a daunting research task.'

There are two aspects to the problem of scale. The first, pointed out by Dooge (1989), is that different physical laws may dominate at different scales. The appropriate set of laws can then be chosen in the light of the scale and type of the problem to be solved. He suggests the following classification of the range of problems in hydrology (Table 1):

Table 1. Spatial Scales in Hydrology

Class	System	Typical Length (metres)
macro	large catchment	10^5
meso	small catchment	10^4
	sub-catchment	10^3
	module	10^2
micro	representative elementary volume	10^{-2}

The second aspect of scale is defined by Miller (1980). The analysis of general problems in applied physics should begin with their expression in the smallest possible number of reduced variables. Any solution for a problem that has been worked out in reduced formulation holds true for an infinite variety of actual systems which, although they differ physically, are simply scale models of each other. Such systems are said to be similar.

Applying this consideration to hydrology, O'Loughlin (1994) observes that rigorously defined criteria for similarity of behaviour between catchments have never been defined. Given the infinite variety of topographic, climatic, soil and vegetational characteristics of catchments, when should we account for the role of topography in redistributing water in the landscape? When can we say that catchments will exhibit similar behaviour?

1.2 Possible approaches

Dooge (1986) observes that while it has been possible to adapt equations from fluid mechanics to solve problems in hydraulics, in moving from the hydraulic scale to hydrologic scale the same approach may be possible but has not yet been achieved. If results are to be obtained at the catchment scale that contribute toward developing hydrologic laws, then the scientific method must be followed. He makes the following suggestions for doing this (Dooge 1986, 1989):

(a) The first step must be the generation of plausible hypotheses that can be tested. One group of such hypotheses can be developed by attempting to combine the nonlinear equations describing hydrologic processes at a continuum point with simple assumptions concerning the microscale parameters. It may be possible to simplify the models of the microscale processes without greatly reducing the predictive power of the resulting mesoscale model.

(b) Another group of hypotheses for catchment scale can be generated by working down from the macroscale by disaggregation of global scale relationships relating to soils, vegetation, drainage networks, rainfall patterns, etc. In this case also, a start should be made with simple hypotheses and then testing them.

(c) When constructing a meso-scale model of catchment response, the individual modules should be as simple as possible, to make the parameters determined during calibration at the meso-scale more reliable. After solving the problem in the simplest possible form we may then proceed to the more complex problem.

This paper examines various approaches to the question raised by Dooge (1986) and O'Loughlin (1994): What laws define the similarity between the behaviours of different hydrologic systems, such as catchments? If such laws are discovered, we can use data relating to particular behaviour (e.g. flood frequency) in one catchment and apply it to predict the behaviour of another catchment of different size. Dooge's (1989) recognition of a range of different scales, possibly involving different laws, will be kept in mind.

An earlier review of scale issues in hydrological modelling has been prepared by Bloschl and Sivapalan (1995). It considers a range of scale issues and has a special focus on modelling. It includes a comprehensive bibliography. The present review differs from theirs in that it adopts a different perspective; it is specifically concerned with the scaling of hydrologic phenomena in real catchments and examines these in order of increasing complexity. It is not particularly concerned with modelling. It looks at what has been achieved so far and what work lays the foundation for future scaling of hydrologic phenomena. Some of the work examined is not directly on scaling but is included because it has, or may have, implications for further work on scaling in hydrology.

2. SCALING PROCEDURES

If the behaviour of a physical system can be described by a set of interrelated dimensional quantities, then these can be transformed into a set of non-dimensional quantities that conserve the original relationships for the system. If two systems (e. g. catchments) are similar, then the corresponding non-dimensional quantities for each of the systems must be equal. Tillotson and Nielsen (1984) distinguish between two different approaches used to obtain the nondimensional quantities: dimensional analysis and inspectional (or similitude) analysis.

2.1 Dimensional and inspectional analysis

In dimensional analysis, the number of nondimensional terms needed to completely define the physical system is obtained from the Buckingham Pi

Theorem (Buckingham 1914). If n dimensional quantities define a system, then $q = (n - m)$ nondimensional terms are also sufficient to define the system, where m is the rank of a $(p \times n)$ matrix formed using p fundamental quantities. In general, the rank of the matrix equals the number of fundamental quantities.

Several procedures are available for obtaining nondimensional terms. For example, all quantities on which the system depends, and their dimensions, are first listed. A subset of m quantities (called the repeating variables) are chosen, from which all the others can be derived. These must collectively contain all the fundamental quantities and must not combine to form a dimensionless group. The remaining quantities are then cast in nondimensional form by multiplying by the appropriate combination of the repeating variables.

Inspectional (or similitude) analysis, unlike dimensional analysis, requires that the physical laws governing the system be known. The physical laws and the initial or boundary conditions are normalized, i. e. reduced to nondimensional form, while eliminating as many physical constants and variables as possible. The nondimensional terms are obtained by inspection from the nondimensional equations governing the system.

Kline (1986) specifies two steps for the normalization: (a) Make all the variables nondimensional in terms of the appropriate scales of the problem. (b) Divide through the equation by the coefficient of one term to make the equation dimensionless term by term.

2.2 Functional normalization

Tillotson and Nielsen (1984) describe an additional (third) method used to determine scale factors, i. e. the conversion factors which relate characteristics of one system to corresponding characteristics of another. This method, called functional normalization, is an empirical method based on least squares regression analysis. The objective is to coalesce all relationships in the set into a single reference curve that describes the set as a whole. The normalization procedure determines one scale factor for each relationship in the set. This scale factor relates any particular relationship to the reference curve.

However, Tillotson and Nielsen (1984) warn that scale factors defined by regression parameters are conversion factors which empirically relate properties in two systems. On the other hand, scale factors obtained through experimentally verified dimensional or inspectional analysis are conversion factors that have definite physical meaning for the system being studied.

Both dimensional analysis and similitude analysis are relevant to scaling in hydrology. Although, in general, we will not have equations that define all of the physical laws, theoretical considerations will often enable us to group appropriate quantities in nondimensional form. In this case, the approach may be closer to similitude analysis than dimensional analysis.

3. SCALING OF SOIL-WATER PHENOMENA

A number of studies of soil-water phenomena have been carried out at field scale. The areas involved ranged from around 10 ha to 150 ha.

Miller (1980) describes a method (developed by Miller and Miller (1956)) for scaling soil-water phenomena using similitude analysis. He points out that one cannot get very far in this problem just using dimensional analysis. Most of the reduced variables which are developed from similitude analysis contain various combinations of a microscopic characteristic length, λ, representing particle size and a macroscopic characteristic length, L, representing soil profile depth. Both being lengths, λ and L are indistinguishable in dimensional analysis but are easily separated in similitude analysis.

He derived the following dimensionless quantities:

pressure: $p^* = (\lambda/\sigma)p$ (1)

hydraulic conductivity: $K^* = (\eta/\lambda^2)K$ (2)

velocity: $v^* = (\eta/\sigma)(L/\lambda)v$ (3)

time: $t^* = (\sigma/\eta)(\lambda/L^2)t$ (4)

where σ = surface tension;

η = viscosity.

It is assumed that the different media being scaled are statistically similar and that in terms of the scaled pressure, p^*, all similar media exhibit identical static moisture characteristics, $\theta(p^*)$, where θ is the volumetric water content (water volume/soil volume).

3.1 Scaling of matric head and hydraulic conductivity

Using the above dimensionless quantities, Warrick (1990) points out that for a given water content the matric heads, h, and the unsaturated hydraulic conductivities, K, for similar systems (differing only in their microscopic characteristic length, λ) are related as follows:

$$\lambda_1 h_1 = \lambda_2 h_2 = \lambda_{av} h_{av} \tag{5}$$

$$K_1/\lambda_1{}^2 = K_2/\lambda_2{}^2 = K_{av}/\lambda_{av}{}^2 \tag{6}$$

These two equations were used to reduce large volumes of data into average relationships using scaling factors to characterise individual sites (Warrick et al. 1977). The data were thus coalesced into meaningful averages while preserving variability through a set of site-specific factors. Because the soils did not have identical values of porosity, the above equations were assumed to apply for equal degree of saturation (water content/saturated water content) rather than equal volumetric water content. Some of the results are summarised as follows by Dooge (1982):

Warrick et al. (1977) 'found that, by the use of scaling, the sum of the squares of the deviations of predicted soil suction for the 20 field locations (within 150 ha) could be reduced to 15% of its original value and that the scatter in the predicted unsaturated hydraulic conductivity could be reduced to 14% of its original value (Figure 1). The values of the scaling factors were determined by an iterative optimisation technique. The analysis was also applied to 36 sites randomly selected within an area of 87 ha measured by Coelho (1974) and 8 sites, all within 7.2 km, by Keisling et al. (1977).'

3.2 Scaling of infiltration

Sharma et al. (1980) carried out infiltration tests at 26 sites (comprising 618 observations) in a 9.6 ha watershed. The data for each test were expressed in the form of Philip's two-parameter equation (Philip 1957):

$$I = S\,t^{1/2} + A\,t \tag{7}$$

where I = cumulative infiltration, t = time, and S and A are the two parameters. Using Miller's (1980) concepts, they argue that the sorptivity, S, and the parameter, A, can be scaled in the following manner:

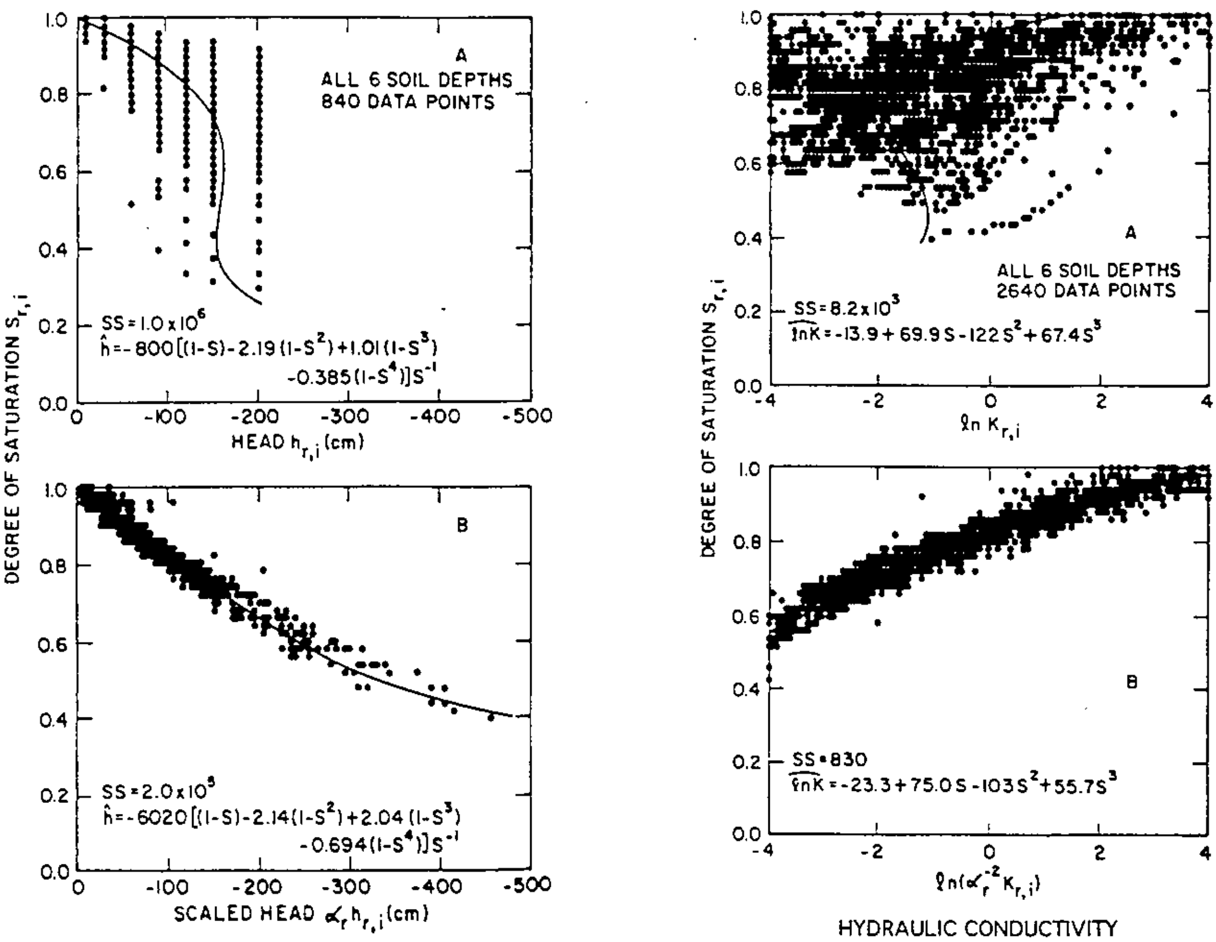

Figure 1. Two examples of soil water characteristic data: above unscaled, below scaled. (Source: Warrick et al. 1977)

$$S_i/\lambda_i^{1/2} = S_r/\lambda_r^{1/2} \tag{8}$$

and

$$A_i/\lambda_i^2 = A_r/\lambda_r^2 \tag{9}$$

where i and r refer to the ith sample and a reference soil respectively. For convenience, dimensionless scaling factors, α, can be defined as:

$$\alpha_i = \lambda_i/\lambda_r \tag{10}$$

Sharma et al. (1980) also define the following dimensionless parameters:

$$\beta = A\,I/S^2 \tag{11}$$

and

$$\tau = A^2\,t/S^2 \tag{12}$$

and write the transformed Philip equation as:

$$\beta = \tau^{1/2} + \tau \tag{13}$$

When all the data points were plotted on β and τ axes, a remarkably good fit to this equation was obtained (Figure 2).

The measured infiltration, $I(t)$, can be scaled using α, based on either S or A, according to the relationship:

$$I^* = \alpha I \quad \text{and} \quad t^* = \alpha^3 t \qquad (14), (15)$$

where I^* and t^* are the scaled cumulative infiltration and scaled time respectively.

Sharma et al. (1980) plotted scaled infiltration against scaled time, using two different least squares estimates of the scaling factor, α, one based on S and the other on A (Figure 3). The scatter of the data points was greatly reduced by scaling. However, the two estimates of α turned out to be significantly different.

Tillotson and Nielsen (1984) point out: 'A direct consequence of the idea that scale factors obtained through functional normalization are not necessarily related to dimensional scaling theories is the fact that a one-to-one

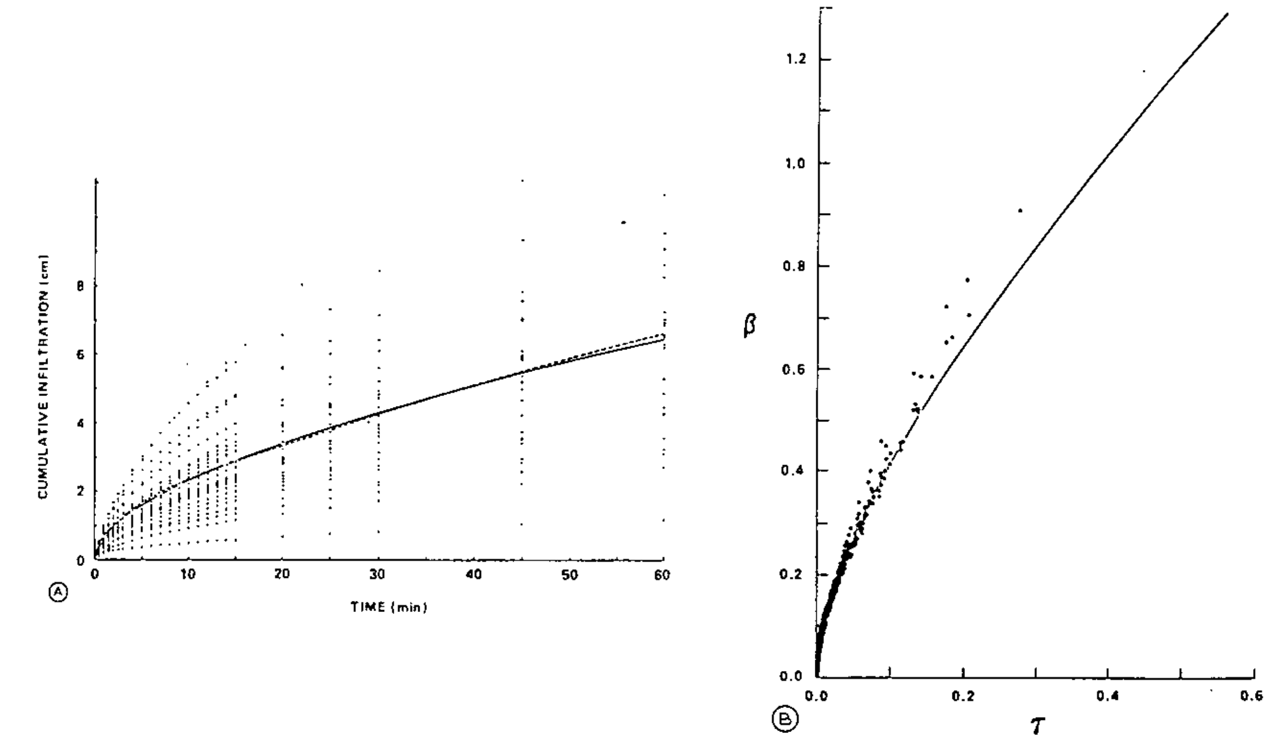

Figure 2. Field measured infiltration versus time: left unscaled, right in dimensionless form. (Source: Sharma et al. 1980)

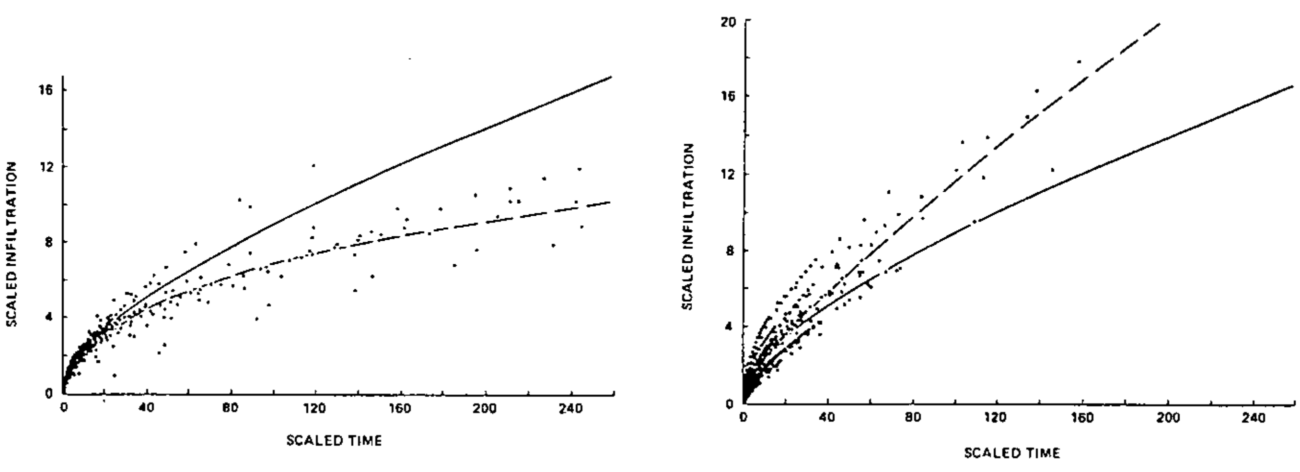

Figure 3. Infiltration data, scaled by α(S) left and α(A) right. Solid lines are computed from average S and A. Broken lines are least squares fits. (Source: Sharma et al. 1980)

correspondence among scale factors determined from various soil properties is not required. In fact the relationship between scale factors obtained through functional normalization for two different soil properties is most likely a mathematical artifact generated by the least squares methods used to obtain them.' The results of Sharma et al. (1980) 'clearly demonstrate that scale factors determined from regression parameters are empirical in nature and should not be assigned physical significance on the basis of scale factors obtained through dimensional techniques.' Similar results were obtained by Warrick et al. (1977) for scale factors determined from hydraulic conductivity and soil water pressure head.

3.3 Concept of similar state

Sposito and Jury (1990) challenge Miller's (1980) postulate that porous media in similar states should have the same volumetric water content. They point out that experimental tests of this postulate, although limited in number, indicate that it describes well-sorted sands reasonably well, but applies poorly to soils containing a broad range of particle sizes.

They argue that the concept of similar state does not require the postulate of invariance of the volumetric water content under scaling of the matric potential. A valid alternative is to postulate the invariance of the pore-size distribution. This involves a different physical interpretation of the scale factor, λ. If the water content is invariant, λ reflects the geometric arrangement of both the pore space and the solid particles. On the other hand, if the pore space distribution is the invariant quantity, then λ is associated only with the pore space, and the solid particle arrangement may have a different scale factor. (The pore-size distribution is measured by determining the relative water saturation as a function of the matric potential.)

The methodologies used in the above examples of scaling of field-scale phenomena may well have relevance also to the larger sub-catchment scale. This would be in accord with Dooge's (1986) suggestion that it may be possible to simplify the models of the various microscale processes and the variation of the microscale parameters without greatly reducing the predictive power of the resulting mesoscale model.

4. SIMILARITY OF CATCHMENT RESPONSES AFTER DISTURBANCE

Suppose a catchment has experienced a disturbance (e. g. reafforestation or clearing). The timescale of response to that event is an important problem in scaling. O'Loughlin (1994) argues that the response time, T, for soil moisture content at any location in the hillslope following disturbance will be a function of the following parameters:

$$T = f(L/KS, b, smd/\Delta q) \qquad (16)$$

where
L = slope length;
K = hydraulic conductivity;
S = slope;
b = vegetation growth rate (% per day);
smd = soil moisture deficit (mm);
Δq = change in evapotranspiration (mm/day).

T and the parameters could be made dimensionless by dividing by some characteristic time for the catchment.

O'Loughlin comments that in any specific case, the form of the relationship will be impossibly complex. It is important then (in any particular problem) to ascertain which of the processes/parameters is the most important and which (if any) can be neglected.

A simple preliminary analysis of the topography throws light on this question. He argues that similarity criteria can find use in delineating land units which will exhibit generally similar behaviour and in segregating parts of landscapes that must be analysed separately because they will exhibit grossly dissimilar behaviour when disturbed. This represents a possible approach to various other scaling problems as well.

5. SCALING OF SURFACE SATURATION ZONES

5.1 Steady state conditions

Storm runoff in catchments depends on soil properties and topography. O'Loughlin (1981, 1986) derived criteria for the existence of saturated areas on hillslopes in catchments and developed dimensionless parameters defining similarity of saturation regions betweeen catchments. He points out that an individual catchment responds to a storm event in a way that is closely related to the prevailing wetness state of the landscape. Local saturation occurs whenever the drainage flux from upslope exceeds the capacity of a soil profile to conduct that flux. He developed the following criterion for local surface saturation in a hillslope under steady state conditions, in dimensionless form (O'Loughlin 1986):

$$w = \frac{1}{MbL}\left(\frac{\overline{T}}{T}\right)\int \frac{q}{\overline{q}} dA \geq \frac{\overline{T} A_t}{Q_0 L} \tag{17}$$

where
M = surface gradient;
b = length of element of contour;
A = partial catchment area;
q = drainage flux;
T = soil transmissivity;
$\overline{T}$ = mean catchment transmissivity;
A_t = total catchment area generating the outflow;
Q_0 = outflow;
$\overline{q}$ = Q_0/A_t;
L = reference length (e. g. mean hillslope length);
w = a wetness function at a point in the catchment.

Using storm records for one catchment, O'Loughlin (1986) plotted runoff ratio (quickflow/total runoff) against $1/Q_0$ (taking Q_0 as baseflow preceding each storm). He superimposed on this diagram the curve showing the percentage area of saturated zone against w, derived from a topographic analysis (Figure 4). A value of $\overline{T}$ was chosen to make the curve form a lower envelope to the data. However, this value is 5 times larger than in situ measurements would indicate and O'Loughlin suggests that these measurements may not adequately

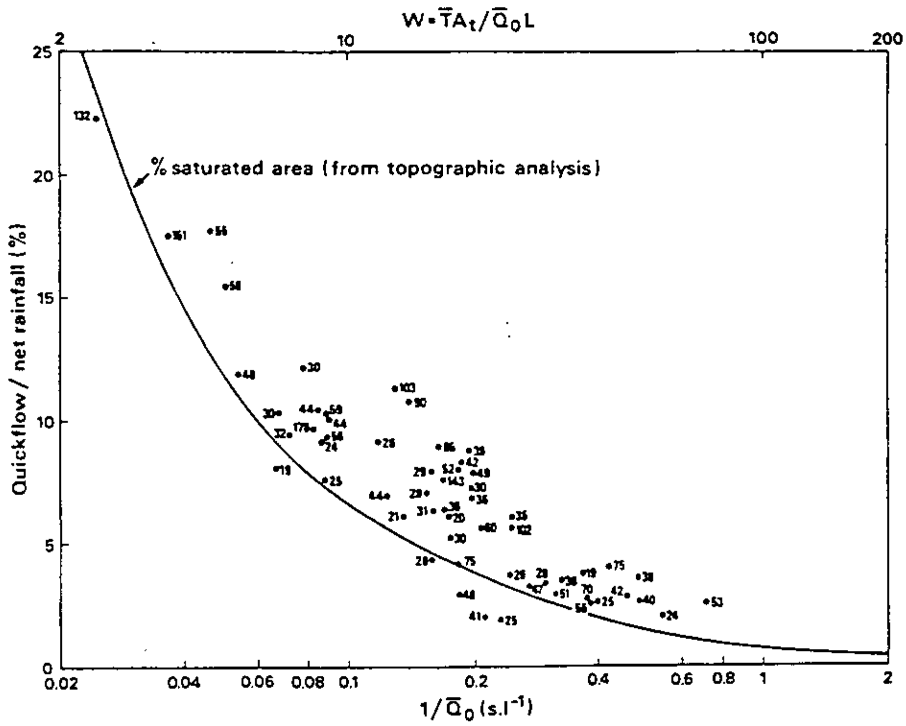

Figure 4. Comparison of saturated area predicted from topographic analysis with actual runoff percentages from recorded storms. (Source: O'Loughlin 1986)

describe the phenomenon. He also argues that the scatter of points above the curve demonstrates that some runoff must be produced by mechanisms other than surface flows from areas that were saturated before each storm.

For uniform T and q, the above criterion for local surface saturation (17) can be written as:

$$A_s / M \geq \overline{T} / \overline{q} \qquad (18)$$

where A_s = partial catchment area per unit contour width. The ratio, A_s/M, is known as a topographic index or a wetness index.

5.2 A quasi-dynamic wetness index

Barling et al. (1994) investigated the distribution of zones of surface saturation under non-steady-state conditions, i. e. before steady state has been reached. They argue that generally the velocity of subsurface flow is so small that most points on a catchment receive contributions from only a small portion of their total upslope contributing area and the subsurface flow regime is in a state of dynamic non-equilibrium.

They developed an alternative, quasi-dynamic wetness index, A_e/M, where A_e = the area of catchment per unit width contributing runoff to the discharge point at a given time. Prior to saturation, $A_e < A_s$. They also compared this index with a third one, developed by Beven and Kirby (1979) and used by Sivapalan et al. (1987), $\ln(A_s/M)$, which corresponds to an exponential relation between subsurface flow and soil moisture storage or to a hydraulic conductivity decreasing exponentially with depth.

Using a computer model, which calculates soil water redistribution and runoff on a continuous basis (Grayson et al. 1992), Barling et al. (1994) simulated a number of rainfall scenarios for a particular catchment. The depth of the perched water table divided by the soil profile depth was taken as a measure of the soil water content. In each simulation, this 'depth ratio' was plotted against each of the three wetness indices. For the case of a continuous rainfall event they found a strong correlation between the index A_e/M and the depth ratio (Figure 5), but no significant correlations between the depth ratio and the A_s/M or the $\ln(A_s/M)$ indices.

Barling et al. (1994) indicate that their theory does not apply to situations where soil piping plays an important role in the hydrologic response of the catchment. In such cases the wetness indices are not good predictors (Jones 1987). Ward and Robinson (1990) describe runoff mechanisms (other than pipe flow) in which each input of rainfall could be accompanied by a virtually instantaneous outflow of subsurface water at the slope foot. This requires the available moisture storage capacity within the system to be already filled. The methodology of Barling et al. (1994) could also be applied to such wet antecedent conditions.

It would be interesting to convert the quasi-dynamic wetness index, A_e/M, to non-dimensional form and apply it to empirical data from different catchments, to determine its potential for scaling zones of surface saturation. It would be necessary to find a way to obtain values of time in a consistent manner, as in reality series of blocks of rainfall superimpose their effects.

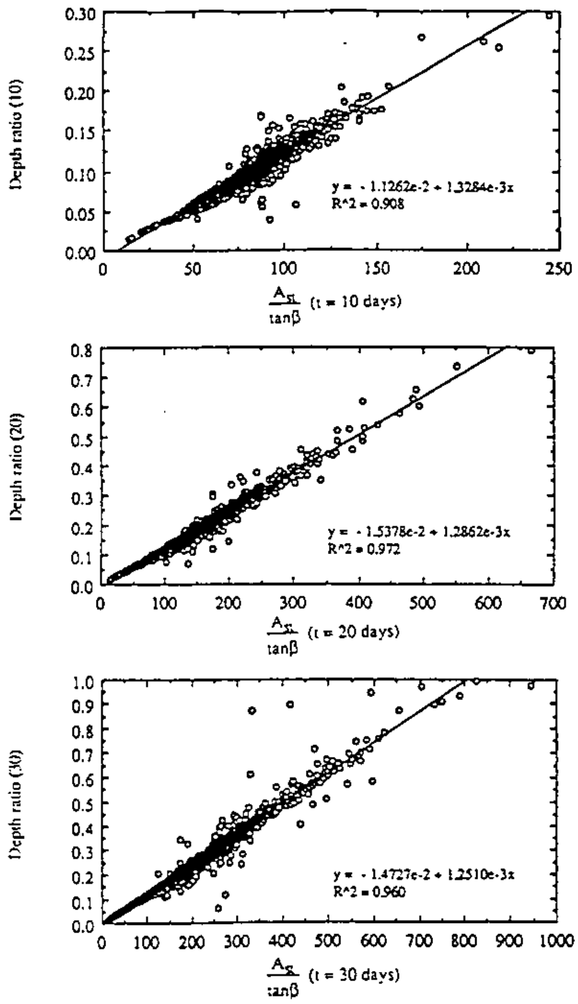

Figure 5. Depth ratio versus the A_e/M index at drainage times of (top) 10, (middle) 20 and (bottom) 30 days. (Source: Barling et al. 1994)

6. RUNOFF GENERATING PROCESSES AND FLOOD FREQUENCY

In recent years a number of papers have explored scaling in runoff generation processes and in flood frequency distributions.

Wood and Hebson (1986) developed similarity relationships for flood frequency distributions that are independent of basin scale. They used the geomorphic unit hydrograph model of Rodriguez-Iturbe and Valdes (1979), assuming a triangular instantaneous unit hydrograph. Their derived dimensionless flood frequency distribution is a function of three dimensionless parameters:

a geoclimatic rainfall scaling factor that defines the areal rainfall distribution;

Horton's length ratio (Horton 1945); and

the ratio of average storm duration to characteristic basin response time.

Sivapalan et al. (1987) developed a scaled model of storm runoff generation due to spatially variable rainfalls on heterogeneous catchments, taking account of the effects of catchment topography on the within-storm dynamics of runoff contributing areas. Their model is based on the earlier work of Beven and Kirby (1979) and Beven (1986).

6.1 A dimensionless flood frequency model

Sivapalan et al. (1990) then developed a dimensionless flood frequency model using a generalised geomorphic unit hydrograph and partial area runoff generation. This utilises the work of Wood and Hebson (1986) and Sivapalan et al. (1987). They define a number of dimensionless similarity parameters. Some of the main ones used in this paper are:

$$p^* = \frac{\bar{p}\tau_r}{\psi_c(\theta_s - \theta_r)} \quad \text{(scaled rainfall intensity)} \qquad (19)$$

$$K^* = \frac{K_0\tau_r}{\psi_c(\theta_s - \theta_r)} \quad \text{(scaled hydraulic conductivity)} \qquad (20)$$

$$Q^* = -\frac{\ln(Q(0)/Q_0)}{\lambda - \mu} \quad \text{(scaled initial soil wetness)} \qquad (21)$$

$$t^* = t_r/t_l \quad \text{(scaled storm duration)} \qquad (22)$$

$$A^* = A/L^2 \quad \text{(scaled catchment area)} \qquad (23)$$

where $\bar{p}$ = areal average rainfall intensity;
τ_r = mean duration of storms;
τ_l = characteristic basin lag time;
θ_s = saturation moisture content;
θ_r = residual moisture content;
ψ_c = depth of capillary fringe;
K_0 = average hydraulic conductivity at surface;

$Q(0)$ = catchment base flow;

Q_0 = a base flow parameter;

$\lambda = \frac{1}{A}\int \ln(a/\tan\beta)dA$;

A = area;

$(a/\tan\beta)$ = a topographic wetness index;

μ = a location parameter in a gamma distribution of $\ln(a/\tan\beta)$;

L = rainfall field correlation length.

The authors plotted p^*/K^*, t^*, Q^*, average contributing area, and (saturation excess runoff/total runoff) against return period for various sets of stochastically derived rainfall and soils data. They found that for a catchment dominated by infiltration excess runoff, the flood frequency curve is determined by the ratio of rainfall to soil hydraulic conductivity (p^*/K^*) and the scaled catchment area (A^*), while initial catchment wetness (Q^*), storm duration (t^*) and average contributing area are fairly constant across return periods (Figure 6). The ratio, saturation excess runoff/total runoff, declines with return period.

For a catchment where saturation excess storm production dominates at low flood return periods and infiltration excess dominates at high return periods, t^*, Q^*, average contributing area and p^*/K^* vary with the return period.

6.2 Testing some parameters

In a further development of this work, Larsen et al. (1994) define another two parameters:

$$f^* = \psi_c f/\lambda \tag{24}$$

$$R = \frac{\int Q_{ser} dz}{\int Q_{tot} dz} \tag{25}$$

where the coefficient f is defined by

$K(z) = K_0 \exp(-fz)$, the hydraulic conductivity at depth z;

Q_{ser} = runoff generated by saturation excess;

Q_{tot} = total runoff.

Note that f^* is not dimensionless (because λ is not dimensionless).

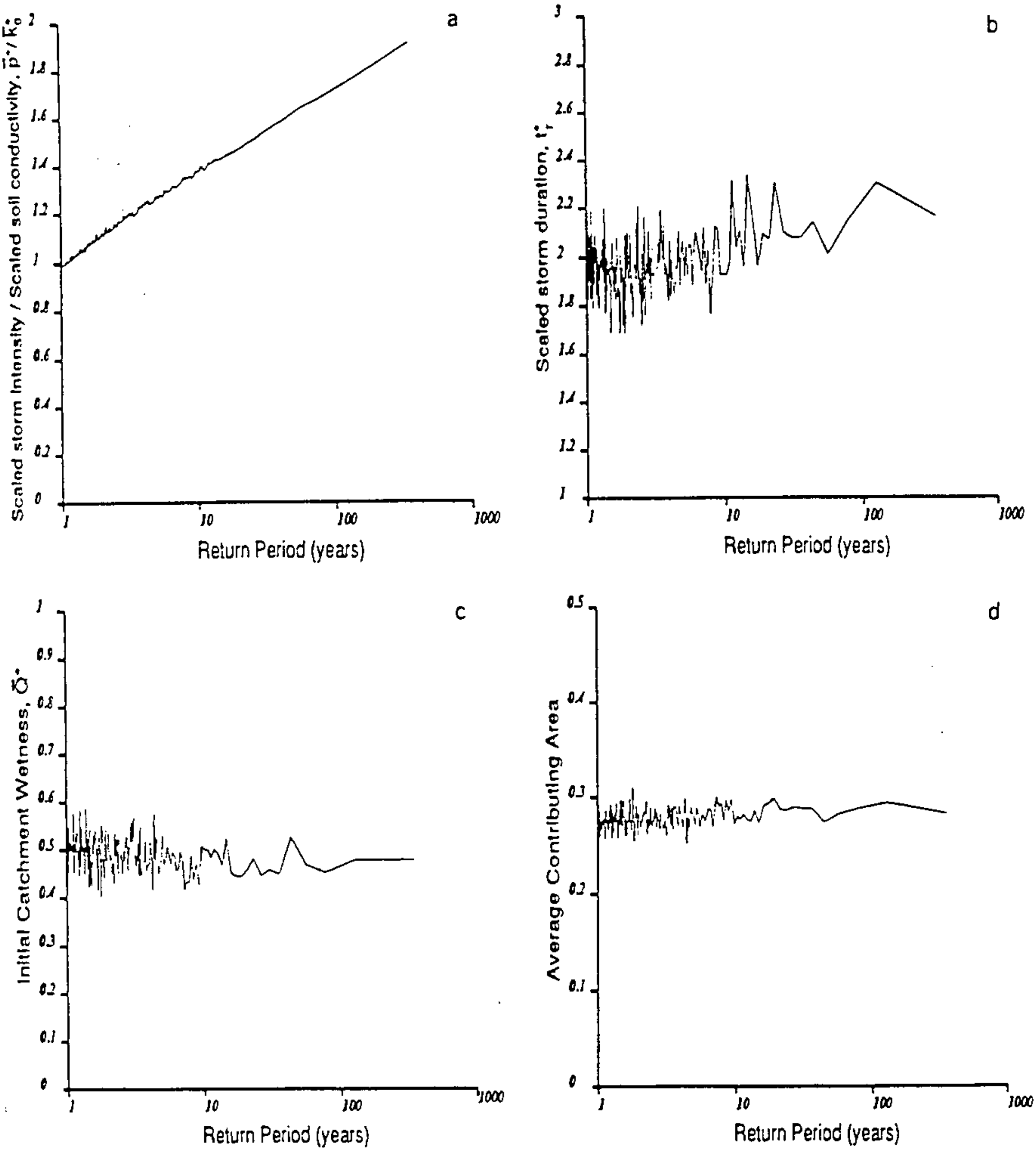

Figure 6. Values of selected parameters of runoff generation model plotted against discharge return period. (Source: Sivapalan et al. 1990)

Using empirical data from a number of catchments, the authors plotted a graph of f^* against K^* together with computer generated contours of R (Figure 7). The actual R values for the catchments come reasonably close to the contours. They argue that this confirms that the two parameters K^* and f^* can account for most, if not all, of the variability of runoff responses between catchments and therefore that two catchments in the region are similar, in terms of their runoff generation responses, if these two parameters are identical.

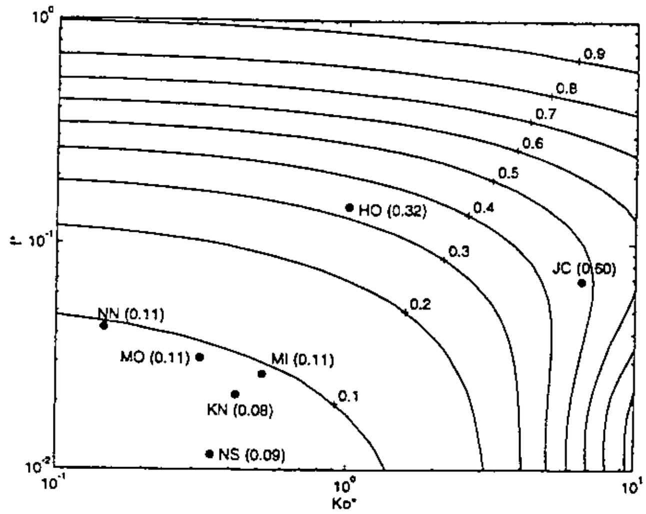

Figure 7. Contours of constant R values as a function of K^* and f^*, together with points (and corresponding R values) for actual catchments. (Source: Larsen et al. 1994)

6.3 Discussion of the model

To what extent does the work reviewed in this section throw light on the problem of scale? The plots of the various dimensionless parameters by Sivapalan et al. (1990) provide insight into the processes involved in the flood frequency distribution (including the significance of different runoff mechanisms). However, some plots of empirical data will be needed before we can confirm the usefulness of these parameters for scaling.

Moreover, it would be worth investigating if some other (perhaps simpler) parameters for rainfall intensity, hydraulic conductivity, initial soil wetness, etc., might be just as useful. For example, does the expression for scaled initial soil wetness, Q^*, really have to be so complicated? And does the variability and uncertainty of the soil properties warrant the use of such terms as θ_s, θ_r, λ and μ?

Can the methodology and the dimensionless parameters used in this modelling investigation be applied to empirical data? To find an answer, it would be useful to plot flood frequency data from a number of catchments using the above parameters and perhaps also some alternative parameters corresponding to a simpler physical model.

In the plot of f^* against K^* by Larsen et al. (1994) for various catchments, only seven points are plotted. The parameter f^* is not dimensionless. It would be interesting first to convert it to a dimensionless parameter and then plot the results for a larger number of catchments. However, the parameters, K^* and f^*, are both measures of the soil hydraulic conductivity (its surface value and rate of decline with depth). The physical significance of the R contour plots on these two axes is not clear.

A number of questions may also be asked about the constant f (the exponential rate of decrease of hydraulic conductivity with depth) used in the parameter f^*. Given the great variability of soil hydraulic conductivity, how can we choose a value of f for a whole catchment? Sivapalan et al. (1990) suggest that f can be estimated for the catchment by analysis of the baseflow recession curve just prior to the storm. Yet how appropriate is the exponentially decreasing model of hydraulic conductivity in a real catchment and how meaningful is a catchment value of f?

Sivapalan et al. (1987) argue that Beven (1982) has provided evidence to show that exponential decrease in conductivity is a reasonable assumption for a wide range of soils. In fact Beven (1982) obtained good correlations when fitting exponential curves to soil conductivities obtained at some sites. (He also obtained good correlations using a power curve.) However, this does not indicate a physical law. There are obviously many soil profiles where an exponential fit would not be successful, e. g. in duplex soils. Furthermore, in real catchments, soil profiles vary from one location to another; there are also tunnels and pipeflow phenomena (Ward & Robinson 1990). Might not a simpler model of hydraulic conductivity be just as useful? For example, could the variation of hydraulic conductivity with depth be covered just as well by a single parameter for transmissivity?

7. GEOMORPHOLOGIC STRUCTURE OF HYDROLOGIC RESPONSE

7.1 Instantaneous unit hydrograph

Rodriguez-Iturbe and Valdes (1979) attempted to link the instantaneous unit hydrograph (IUH) with the geomorphic properties of a basin. This attempt has been taken further by Gupta et al. (1980; 1983). To what extent is this relevant to the problem of scale, in our sense of defining similarity between the hydrologic behaviour of different catchments?

Using a lengthy analysis involving Markov processes, Rodriguez-Iturbe and Valdes (1979) expressed the IUH as a function of Horton's numbers, R_A, R_B and R_L (Horton 1945), a streamflow velocity, v, and a scale parameter, L. They developed equations for the flood peak, q_p, and the time to peak, t_p, which in turn depend on these five parameters. They found that the dimensionless product, $q_p.t_p$, is independent of v and L. It is a characteristic constant for each basin, independent of the storm characteristics and intimately linked to the geomorphology of the watershed and to its hydrologic response structure.

Using a simpler statistical procedure, not involving Markov processes, Gupta et al. (1980) also developed a representation of the IUH from basin geomorphology. A comparison of the theoretical results with observed flows showed good agreement for two of their three case studies.

7.2 An analytical channel model

Gupta and Waymire (1983) argue that the previous two approaches are inadequate and distort the nature of the hydrologic response. In particular, the formulation of the network geometry in terms of the Strahler ordered channels (Strahler 1964) was inappropriate. Instead, Gupta and Waymire attempted the formulation of an analytic approach to hydrological response that involves an analytical channel model, building on the work of Shreve (1966). They describe their model as theoretical in the sense that it aims at deducing observed empirical laws within a mathematical framework.

They formally define the hydrologic response of a channel network as the time history of the arrival of particles at the outlet for an initial uniform distribution of particles injected simultaneously at each source and each junction of the tree and then allowed to travel with the same constant velocity along the links to the outlet. They define channel networks as similar with respect to their response functions if they have the same response functions after these have been transformed to a common time scale. In that case they will also be similar with respect to peak flow, time to peak, and mean lag time (when the last two are transformed to a common time scale).

As an example, they considered the problem of hydrologic similarity with respect to mean lag time amongst all networks having a fixed magnitude, M (i. e. having M sources). They derived a mathematical expression for the mean lag time for each network, which turned out to be in reasonable agreement with an empirically-based formula. However, in general they conclude that although they have introduced the key variables of a channel network that determine its response function, no mathematical framework has been identified to derive the structure of the interrelationships of the variables.

To what extent is the work reviewed in this section relevant to our task of scaling, in the sense of defining similarity between the behaviour of different catchments? The work of Rodriguez-Iturbe and Valdes (1979) and Gupta et al. (1980) provides much insight into the geomorphic basis for hydrologic responses as expressed in the IUH. The work of Gupta and Waymire (1983) defines and examines the similarity of basin response functions, but the lack of an adequate mathematical framework means that it cannot be used as a basis for scaling between catchments at this stage. However, the concepts behind the model are important for future work and have led to the developments considered in the next section.

8. SIMPLE SCALING AND MULTISCALING IN RIVER NETWORKS

The past ten years have seen the development of the mathematical theory of simple scaling and multiscaling, which is now being applied to some hydrologic phenomena. This section will examine this development.

Gupta et al. (1986) point out that a basin is made up of two interrelated systems: the hillslopes and the channel network. They argue that to explore the linkage between the two systems the vertical dimension must be taken into account. However, this third dimension has been largely ignored in previous theory. In particular, the drops in elevation among the channel links of the network should be connected through some unifying principles. (A link is a portion of the channel between two nodes.)

8.1 Scaling invariance

Gupta and Waymire (1989) introduce the concepts of scaling invariance (also called simple scaling) and statistical self similarity and apply them to the distribution functions of link heights in river networks. The link height is the elevation difference between the two nodes of a link.

Let the link magnitude, m, be a scale parameter, $\mu(m)$ a 'scaling function', and Z_i a set of random variables not dependent on m. The distribution functions of the link heights, $H_i(m)$, are defined as having scaling invariance if

$$\frac{H_i(m)}{\mu(m)} = Z_i \qquad i = 1, 2, \ldots \tag{26}$$

If λ is a scalar, the link heights, $H_i(\lambda m)$, are defined as self similar if the distribution function of the ratio

$$\frac{H_i(\lambda m)}{\mu(\lambda)} = H_i(m) \qquad i = 1, 2, \ldots \tag{27}$$

is independent of λ for all $\lambda > 0$. (In the above two equations, '=' means having the same distribution.)

Gupta and Waymire (1989) also define a link concentration function (LCF) as a stochastic process parameterised by elevation, h. It represents the number of links at various elevations, h, above the outlet. They plotted the normalised LCFs (the proportion of links versus the elevation) for four basins together with (in each case) theoretical curves based on four different sets of scaling assumptions (Figure 8). They found that the theoretical curve based on their self similar model fitted the empirical data better than the curves based on other assumptions. However, the fits are not particularly close.

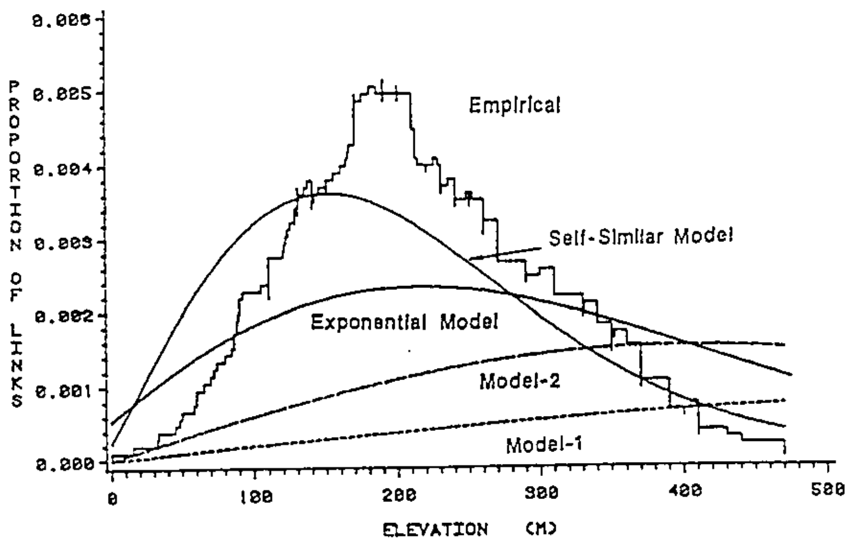

Figure 8. Example of normalized empirical and conditional mean LCFs for 4 different scaling models of link heights for a basin. (Source: Gupta & Waymire 1989)

8.2 Multiscaling of flood peaks

Gupta and Waymire (1990) define another term: multiscaling. Suppose we wish to examine the average rainfall over a pixel of side length λ. Let $m_h(\lambda)$ denote the statistical moment of order h, and consider a plot of

$$\log(m_h(\lambda)) = a_h + \theta_h \log(\lambda) \tag{28}$$

where the intercept a_h and the slope θ_h are estimated by regression. The stochastic process is defined as multiscaling if the above plot is linear for each h while the plot of θ_h against h is non-linear. The authors found that multiscaling occurred in empirical data plots they examined for spatial rainfall, spatial river flows and turbulent velocities.

Smith (1992), Gupta and Dawdy (1995) and Gupta et al. (1994) applied simple scaling and multiscaling theory to the statistical distribution of annual flood peaks, parameterised by drainage area. The analysis of data from a number of catchments by Gupta and Dawdy (1995) suggests that floods generated mainly by snowmelt runoff exhibit simple scaling while rainfall-generated floods exhibit multiscaling. They argue that the multiscaling property of rainfall-generated floods is a consequence of the multiscaling property of rainfall, demonstrated, for example, by Gupta and Waymire (1993).

As an example of the multiscaling of flood peaks, Gupta et al. (1994) plotted the first four statistical moments for a large set of catchment data, together with the lines predicted by multiscaling theory using a lognormal model (Figure 9). A good fit was obtained.

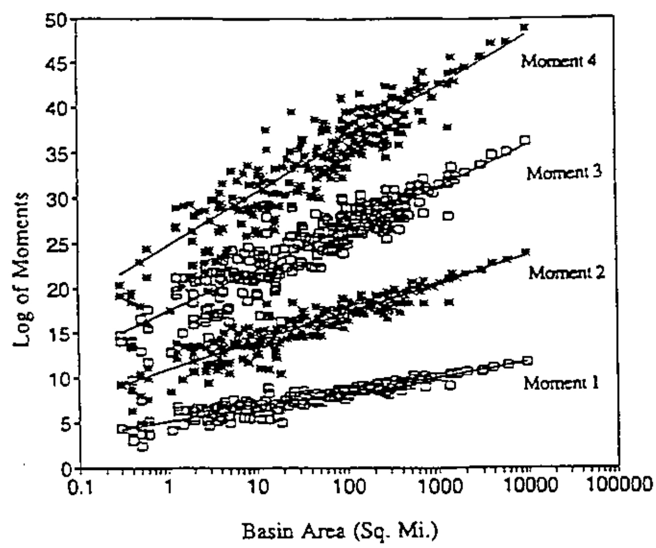

Figure 9. Multiscaling predictions of flood peak moments using a lognormal model. (Source: Gupta et al. 1994)

The presence of multiscaling appears to be much weaker in small basins than in large basins (Gupta et al. 1994). Furthermore, there appears to be a critical basin area, A_c (probably between 50 and 260 km^2) which defines two different ranges of a scale parameter, λ. Let A be the drainage area and A_{max} and A_{min} two extreme values, outside all the drainage areas in the region. For basins with $A > A_c$, $\lambda = A/A_{max} > 1$, and for those with $A < A_c$, $\lambda = A/A_{min} > 1$.

Gupta and Dawdy (1995) suggest that there is a physical basis for the difference in the scaling structure of floods between small and large basins. Hillslope processes and channel network geometry dominate in the response of small basins, while in large basins the response is input dominated, i.e. the spatial variability of precipitation becomes important.

Multiscaling theory is still in process of development. Gupta and Dawdy (1995) note that there are many questions still to be answered for the development of a general theory of flood frequency. The indications are that multiscaling will be very important in the scaling of flood peaks and probably other hydrologic phenomena.

9. CLIMATE, SOIL AND VEGETATION

Eagleson (1978) examined the interrelationship between climate, soil and vegetation. He developed a stochastic-dynamic formulation of the vertical water budget at a land-atmosphere interface.

The average annual water balance for soil moisture is given by:

$$E[I_a] = E[E_{ta}] + E[R_{ga}] \tag{29}$$

where I_a = annual total infiltration;
E_{ta} = annual total evapotranspiration from soil moisture;
R_{ga} = annual groundwater runoff;
$E[\ ]$ = expected value.

Eagleson (1978) assumed representative probability density functions for the interval between storms, duration of storms, rainfall intensity and potential evapotranspiration. He thus obtained detailed expressions (including dimensionless parameters) for each term in the above equation:

> $E[I_a]$ is a function of average annual precipitation, a gravitational infiltration parameter and a capillary infiltration parameter;
>
> $E[E_{ta}]$ is a function of average annual potential soil moisture evapotranspiration, an exfiltration parameter, vegetal canopy density and potential transpiration efficiency;
>
> $E[R_{ga}]$ is a function of mean length of rainy season, saturated hydraulic conductivity, average soil moisture concentration in surface boundary layer and rate of capillary rise.

Eagleson then reduced the annual water balance equation to dimensionless form, involving nine dimensionless parameters which define the conditions for annual water balance similarity.

9.1 Long-term equilibrium

In a further development, Eagleson (1982) suggested the existence of equilibrium relationships between long-term evapotranspiration and the state of the vegetation canopy. Dooge (1989) sums up this work as follows: 'Eagleson (1982) suggested that under conditions of water limitation, a system of vegetation would, for the given climate and soil moisture conditions, produce the particular canopy density which reduced moisture stress at the roots to a minimum. For the case where vegetative activities are limited by energy rather than by water, Eagleson suggested that the vegetative system would tend to maximize the biomass for the given amount of energy. By applying these two hypotheses to his 1978 stochastic-dynamic model, Eagleson (1982) derived the equilibrium relationship defining the limiting curves relating the ratio of actual to potential evapotranspiration (which is species dependent) to the density of the vegetative canopy. Preliminary comparison of data for a few catchments in humid and semi-arid regions tends to confirm the derived limiting relationships as reasonable (Eagleson & Tellers 1982).'

Some aspects of Eagleson's methodology, including the development of the dimensionless parameters, may be useful in the scaling of hydrological phenomena. Of particular importance is his treatment of vegetation, in its interaction with climate and soil.

10. FRACTALS AND SCALING

When certain objects and systems are examined at smaller and smaller scale, the same pattern repeats itself. Such objects and systems are called 'fractals'. They possess the property of 'self-similarity'.

When each piece of a shape is geometrically similar to the whole, it is self-similar, in the strict sense (Mandelbrot 1983). However, in fractals where the small copies look like the whole but have variations we have 'statistical self-similarity' (already considered from another perspective in section 8). There are also objects that exhibit 'self-affinity', in which there is a difference between the scaling behaviour in two different directions (Peitgen 1992).

Self-similar objects do not have a uniquely defined length. Rather, the measured length is a function of the length of the measuring device. A coastline is a good example. The measured length increases indefinitely as the measuring rod gets smaller. For any fractal trace, the general length relation is given by (Tyler & Wheatcraft 1990):

$$L = H e^{1-D} \tag{30}$$

where L is the length, H is a constant, e is the measuring unit, and D is called the 'fractal dimension'.

Fractals have been applied to the analysis of hydrologic systems. Chang et al. (1994) applied fractal theory to describe the fingering structure of water in a soil profile. They carried out several experiments on the infiltration and movement of water in clays and sands. The study demonstrated that the effective surface tension of fingering may be estimated from the fractal dimension, D, the constant, H, and the mean pore size.

10.1 Fractal nature of river networks

Tarboton et al. (1988), Rosso et al. (1991) and Nikora (1994) have examined the fractal nature of river networks. Tarboton et al. (1988) argue that the fractal dimension of the total length of a river network is:

$$D = \log R_B / \log R_L \tag{31}$$

where R_B and R_L are Horton's bifurcation and length ratios (Horton 1945). Examining 9 river basins in the USA, they found that D has a value close to 2.

Rosso et al. (1991) argue that the fractal dimension of the mainstream length of a river is:

$$D = \max(1,\ 2 \log R_L / \log R_A) \tag{32}$$

where R_A and R_L are Horton's area and length ratios. They examined 5 river basins in Italy and 8 in Missouri, USA, and compared measured values of D with values obtained from this equation. Reasonable agreement was obtained.

Nikora (1994) examined the self-similarity and self-affinity of drainage basins. After analysing 25 river basins in various countries, he concluded that river basins are generally self-affine. However, he does not rule out the possibility that the drainage basins exhibit self-similar behaviour over a certain range of scales.

In the above papers, fractal theory was used to describe physical systems. It is a big step to go from there to the use of fractals in the scaling of hydrologic processes. It is possible that future developments may lead to such applications. Furthermore, some aspects of fractal theory are closely related to simple scaling and multiscaling theory, considered in section 8.

11. THE REPRESENTATIVE ELEMENTARY AREA CONCEPT

Wood et al. (1988) pose the question: How does the statistical behaviour of runoff generation change with increases in catchment scale? They hypothesise that at small scales the patterns of topography, soil, and rainfall characteristics are important in governing runoff production. However, as scale increases, more and more of the variability in the distributions is sampled within each area, until eventually at some large scale, all areas will yield almost identical responses for the case of stationary distributions. They suggest that this threshold scale represents a 'representative elementary area' (REA) which will be a fundamental building block for catchment modelling.

Wood et al. (1988) used simulations, based on the topography of the Coweeta River basin in North Carolina (area = 17 km^2), to examine this hypothesis. For each of 5 realizations of rainfall and 3 correlation lengths, they determined the runoff volume for 148 sub-catchments. (Correlation length is a measure of the distance beyond which there is no correlation of the rainfall.) These runoff volumes were ranked on the basis of catchment size, and then mean runoff volume was plotted against mean sub-catchment area. The plots were used to determine the REA, taken as the area where the curve flattened out.

The study concluded that an REA does exist in the context of the runoff generation response of catchments. (Its area was found to be about 1.0 km^2.) The REA was strongly influenced by topography, while the variabilities of soils and rainfall inputs between sub-catchments appeared to have only a secondary role.

Wood et al. (1990) report on similar simulations carried out in another catchment. The ratio of runoff to precipitation depth was examined and the same REA of 1.0 km^2 was found.

Bloschl et al. (1995) showed that the existence of an REA requires a 'separation of scales' or 'spectral gap' in catchment variability (of topography, soil, vegetation and rainfall fields). They carried out another simulation, also using a distributed parameter model, on the same catchment as Wood et al. (1988). They used sets of nested sub-catchments and considered the effects of flood routing.

They found that the size of the REA, when it exists, will be specific to a particular catchment and a particular application. It is strongly controlled by the correlation length of precipitation and also depends on many other factors, including storm duration and variability, flow routing and infiltration characteristics. They consider that there is no evidence for one universal size of an REA or one universal 'optimal element size' in the context of distributed rainfall-runoff modelling. Furthermore, it may be that an REA will generally not exist.

Fan and Bras (1995) argue that since an REA requires a clear separation of scales in the source variability, it is unlikely to exist in a natural environment, for a general separation of scales is not warranted in nature where rainfall, as well as topography and soil, are known to vary at many nested scales. Even in situations where an REA does exist, they argue that it does not provide a definite and robust measure of spatial variability, because it is a function of a range of scales and the individual events examined.

12. CONCLUSIONS

There are two aspects to the problem of scale in hydrology. The first is the recognition that different laws may dominate at different scales. For example, hillslope runoff processes may dominate the response at sub-catchment scale; the channel network geometry becomes more important in meso-scale basins (up to the order of 100 km^2); while in large basins the spatial variability of precipitation becomes very important.

The second aspect of scale involves the establishment of dimensionless parameters for a problem, so that a solution can be applied to an infinite variety of systems. Two different but related procedures are used to obtain sets of dimensionless parameters: dimensional analysis and similarity analysis. Both of these are relevant to scaling in hydrology.

Scaling of soil water phenomena has been carried out successfully at field scale, using dimensionless parameters defined by Miller and Miller (1956). The results show both the value of working rigorously with physical laws and the difficulty of obtaining completely consistent results, even with such relatively simple phenomena. The methodologies provide a partial model for scaling at larger hydrologic scales.

Considerable work has been done on the scaling of runoff generating processes and flood frequency, using computer modelling (Wood and Hebson 1986; Sivapalan et al. 1990) and some plots of catchment data (Larsen et al. 1994). The computer plots of dimensionless parameters provide insight into the processes involved in the flood frequency distribution. However, scaling in a model is a very different process from the scaling of hydrologic phenomena in real catchments and many plots of empirical data will be needed before the usefulness of these parameters for scaling can be confirmed. The parameters involving soil hydraulic conductivity appear to be unnecessarily complex.

Simple scaling and multiscaling theories have been applied to some hydrologic phenomena. In particular, they have been found appropriate for the scaling of flood peaks (Gupta et al. 1994). Analysis of catchment data suggests that floods generated mainly by snowmelt runoff exhibit simple scaling while rainfall-generated floods exhibit multiscaling. Multiscaling theory is in process of development and there are many questions still to be answered. It may prove the key to a practical procedure for the scaling of flood peaks.

Eagleson (1978, 1982) has examined the interrelationship between climate, soil and vegetation, and has developed dimensionless parameters defining the conditions for annual water balance similarity. His methodology, especially for the treatment of vegetation and for the development of dimensionless parameters, may prove useful in scaling.

Fractal theory has been applied to the analysis of some hydrologic phenomena, including river networks. This work does not appear to lead to practical scaling applications at this stage. However, it may in the future, for example through its relation to simple scaling and multiscaling theory.

The representative elementary area (REA) concept, developed by Wood et al. (1988) is not relevant to the task of scaling. Bloschl et al. (1995) have found that the REA, when it exists, will be specific to a particular catchment and a particular application. Fan and Bras (1995) also challenge the existence of an REA in a natural environment.

13. RECOMMENDATIONS

(1) In further research on the scaling of hydrologic phenomena, priorities must be set as to what problems to investigate. The possibilities include: stream discharge, water yield, flow-duration curves, flood frequency curves, peak floods, surface saturation zones, and dynamic response time. A relatively simple problem should be chosen first.

(2) While there has been reasonable success in scaling soil-water phenomena at field scale (approximately 1 km^2), to go from here to working with catchment data on a larger scale is a big step. The difficulties include: (a) the large number of variables and physical laws that govern the phenomena, (b) the spatial distribution of such properties as soil hydraulic conductivity, (c) the stochastic nature of such variables as storm intensity, storm duration and soil moisture condition, and (d) the systematic distribution of catchment attributes (topography, soils, etc.). It is therefore essential that in planning further work, a thorough-going 'systems' approach be used; i.e. the problem must be clearly defined, and appropriate methodologies developed and progressively reassessed after working with data.

(3) The magnitude of scale for each investigation should be determined, e. g. large basin, small catchment, sub-catchment. As Dooge (1989) points out, different laws may dominate at different scales and the appropriate set of laws then chosen in the light of the scale and type of problem to be solved. It is desirable to begin with a fairly small scale and, if successful, to then move up to a somewhat larger scale.

(4) For each problem investigated it is necessary to identify the various quantities that govern the hydrologic behaviour and to identify the physical laws operating. The quantities will include the physical attributes of catchments (soils, topography, vegetation) and the driving climate variables (precipitation, evaporation). The dimensionless parameters must then be established, using dimensional analysis or similitude analysis.

(5) In general, the number of variables governing a phenomenon will be too large for all to be considered and there will be many uncertainties in properties of soils, topography, etc. So it will be necessary to simplify. The initial model should be made as simple as possible. After solving the problem in that form, we may then proceed to the more complex problem.

(6) In considering soil hydraulic conductivity, a simple theoretical model should be used initially, e. g. using the overall transmissivity of the profile.

(7) In any investigation, the dimensionless parameters derived should be evaluated by testing them against historical data sets from catchments where the individual variables (and their dimensionless counterparts) cover a wide range of values.

(8) Bearing in mind the above principles, an appropriate initial study would be the scaling of base flow discharge from a sub-catchment. This is a relatively simple phenomenon, involving a small scale (less than 10 km^2). The discharge is a function of such variables as storm history, catchment area, soil moisture condition, hydraulic conductivity and vegetation state. Steps in the investigation would include: development of a simple theoretical model, definition of dimensionless parameters, assembly of historical data sets, plotting of dimensionless parameters for this data, and further refinement of the model.

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