JOINT PROBABILITY APPROACHES TO DESIGN FLOOD ESTIMATION: A REVIEW

A. Rahman
T. M. T. Hoang
P. E. Weinmann
E. M. Laurenson

Report 98/8
December 1998

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

JOINT PROBABILITY APPROACHES TO DESIGN FLOOD ESTIMATION: A REVIEW

A. Rahman
T. M. T. Hoang
P. E. Weinmann
E. M. Laurenson

Report 98/8

December 1998

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

Rahman, Ataur

Joint probability approaches to design flood estimation: a review

Bibliography.
ISBN 1 876006 40 4.

Flood forecasting – Australia – Mathematical models. 2. Runoff – Australia – Mathematical models. 3. Rainfall probabilities – Mathematical models. I. Rahman, Ataur, 1959- .II. Cooperative Research Centre for Catchment Hydrology. (Series: Report (Cooperative Research Centre for Catchment Hydrology); 98/8).

551.4890112

Keywords

Rainfall/Runoff Relationship
Floods and Flooding
Design Data
Modelling (Hydrological)
Distribution (Mathematical)
Frequency Analysis
Precipitation (Atmospheric)

ISSN 1039-7361

PREFACE

The core projects undertaken in the Flood Program of the CRC were formulated over several sessions of a Technical Advisory Group (TAG), comprised of industry and research representatives. From the process of identifying the projects for which research would have most "return" for industry, the aim to develop a holistic approach to design flood estimation figured highest on the project list.

This report is a review of joint probability approaches to this issue i.e. approaches which consider the combination of factors that produce flood flows. For example, the chances of heavy rain on a dry catchment, or intense rain over part of a catchment, are typical of the variables inputs to be considered.

The work on joint probability is being done in parallel with research on continuous simulation. These can be considered as alternative ways to deal with the infinite number of combinations of factors which can produce the same flood level.

I'd like to take the opportunity to acknowledge the efforts of the authors of this report, and of the work being done in this CRC Project (Holistic Approach to Rainfall-based Flood Estimation) under the leadership of Erwin Weinmann.

Russell Mein
Leader, Flood Program

SUMMARY

Rainfall-based flood estimation techniques are common in hydrologic practice. The currently used methods are based on the *design event approach*; they use a probabilistic rainfall depth in combination with representative values of other inputs and then assume that the resulting flood has the same frequency as that of the input rainfall depth. In many cases, this assumption is unreasonable, and the arbitrary treatment of various inputs is likely to introduce significant bias in flood estimates for a given frequency. The report critically examines the limitations of the current design event approach and identifies the potential alternative methods that might lead to an improved rainfall-based design flood estimation technique.

The serious limitations of the current design event approach stem mainly from the simplifying assumption that with a representative set of inputs and model parameter values, the design flood output will preserve the annual exceedance probability of the design rainfall depth input. While there have been recent improvements in defining more representative design values for losses (Hill et al., 1996a, b) and there is some scope for developing more consistent sets of temporal patterns, even improved sets of single-valued design inputs will not be able to adequately allow for the complex interaction of rainfall and loss parameters with other catchment attributes (e.g. catchment size, shape, drainage characteristics).

The most promising alternatives to the design event approach include the continuous simulation approach (possibly also in its simplified form using runoff files) and the *joint probability approach*. These two approaches are similar in their (deterministic) modelling of the hydrograph formation process (runoff routing), but differ in the form of their basic inputs and how they use these to represent the runoff generation phase. This report focuses on the joint probability approach while the continuous simulation approach is being covered in a parallel report.

The promise of the joint probability approach stems from the fact that it can readily utilise the (deterministic) models and much of the design data used with the current design event approach, but will apply them within an appropriate probability framework (Laurenson, 1974). This framework also exists and only needs to be adapted to this specific application. Joint probability methods therefore have the potential to lead to significant improvements in flood estimation,

with relatively modest effort. The challenge lies in distilling the best elements of the existing methods and design data, and then combining them in a practically useful way to produce the required design tools.

The report presents a review of the previous studies in the area of joint probability approach to design flood estimation. It focuses particularly on the results of previous studies in relation to *practical applicability* of the methods. It has been found that most of the previous applications employing the joint probability approach were limited to theoretical studies; mathematical complexity, difficulties in parameter estimation and limited flexibility preclude these techniques to be applied under practical situations.

This review indicates that rainfall duration, rainfall temporal pattern and losses are key variables to be treated as random variables in addition to rainfall depth. An initial loss-continuing loss model combined with a semi-distributed non-linear runoff routing model (e.g. RORB, URBS) would be appropriate to use with the joint probability approach. From the consideration of practical applicability and ability to account for dependence between the flood producing variables, Monte Carlo simulation and the application of the Total Probability Theorem to discretized distributions appear to be the most promising methods for determining derived flood frequency distributions. This review concludes with a list of research tasks to develop a practical design tool for flood estimation using the joint probability approach.

ACKNOWLEDGMENTS

The authors would like to thank Assoc. Professor Murugesu Sivapalan and Dr. Gunter Bloschl for providing some relevant papers, Dr. Rory Nathan, Dr. Bryson Bates, Dr Sri Srikanthan, and Professor Russell Mein for their comments on the draft report.

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

1. INTRODUCTION

1.1 PURPOSE OF THE REPORT

The purpose of this report is to ascertain the current state of research and practice in joint probability approaches to design flood estimation as a basis for further research with this approach. This is part of the Cooperative Research Centre for Catchment Hydrology (CRCCH) Project FL1: Holistic Approach to Rainfall-based Design Flood Estimation.

The specific objectives of this review are:

- to examine critically the limitations of the current design event approach commonly applied for rainfall-based design flood estimation in Australia;
- to review previous applications of the joint probability approach to design flood estimation with a particular emphasis on practical applicability; and
- to recommend the direction of research towards an improved design flood estimation technique that would overcome the major limitations with the design event approach and could be applied easily in practice.

1.2 BACKGROUND

Estimation of design floods is necessary for planning and design of engineering projects subject to flood risk. Ideally, design flood estimates should be based on the statistical analysis of long streamflow records observed at the catchment of interest. For catchments with little or no streamflow data and where catchment conditions have changed significantly over the period of streamflow record, rainfall-based flood estimation techniques are commonly adopted.

The rainfall-based flood estimation techniques used currently are based on the *design event approach* in that design rainfall intensity for specified durations and annual excedance probabilities (AEPs) are used in combination with "typical values" of other relevant inputs and parameters. It is then assumed that the resulting flood estimate has the same AEP as that of the input rainfall depth. This assumption is generally not satisfied and the arbitrary treatment of various flood producing components can lead to inconsistencies and significant bias in flood estimates for a given AEP. This is likely to result in systematic under- or over-design of engineering structures, both with important economic consequences.

The basic problem in rainfall-based design flood estimation is to find appropriate deterministic models to represent the transformation of rainfall inputs to flood outputs and to preserve the important probability characteristics involved in this process. Two basic approaches have been proposed to address these two requirements in a more holistic fashion than the current design event approach: the joint probability approach and the continuous simulation approach. Both of these form separate components of CRCCH Project FL1. This report deals specifically with the joint probability approach but contrasts it with alternative approaches.

While Australian Rainfall and Runoff (I. E. Aust., 1987), referred to as ARR henceforth, adopted the design event approach to rainfall-based design flood estimation, it recognised the importance of considering the probabilistic nature of the flood producing inputs and their interactions. It thus recommended further investigation into joint probability approaches. More recently, Hill and Mein (1996), in a study of incompatibilities between storm temporal patterns and losses for design flood estimation, mentioned that "a holistic approach will perhaps produce the next significant improvement in design flood estimation procedures".

There is a considerable body of literature that deals with methods to derive a flood frequency distribution from joint consideration of frequency distributions of rainfall and other flood producing factors. Most of these applications of the joint probability approach have not considered the practical limitations of the approach for routine design flood estimation. The purpose of this component of Project FL1 is to find the best elements of existing joint probability methods, deterministic models and design data, and to develop them into a practically useful design methodology.

1.3 OUTLINE OF THE REPORT

The remainder of this report consists of four chapters, as described below.

A critical review of the design event approach is presented in Chapter 2. Alternative methods to the design event approach are then discussed in overview to place the joint probability approach in context.

Previous studies of the joint probability approach for design flood estimation are reviewed in Chapter 3. Important flood producing components and methods for obtaining their probability distributions are also discussed in this chapter.

Chapter 4 contains recommendations for further research to develop a design flood estimation technique based on the joint probability approach that can be used easily in *practice*.

Chapter 5 draws conclusions from this review study.

Appendix A contains the statistical basis of the joint probability approach.

2. RAINFALL-BASED FLOOD ESTIMATION TECHNIQUES

2.1 OVERVIEW OF DESIGN FLOOD ESTIMATION TECHNIQUES

Design flood estimation methods can be broadly classified into two groups: streamflow-based methods and rainfall-based methods (Lumb and James, 1976, Feldman, 1979, James and Robinson, 1986, I. E. Aust., 1987). This classification is illustrated in Figure 1.

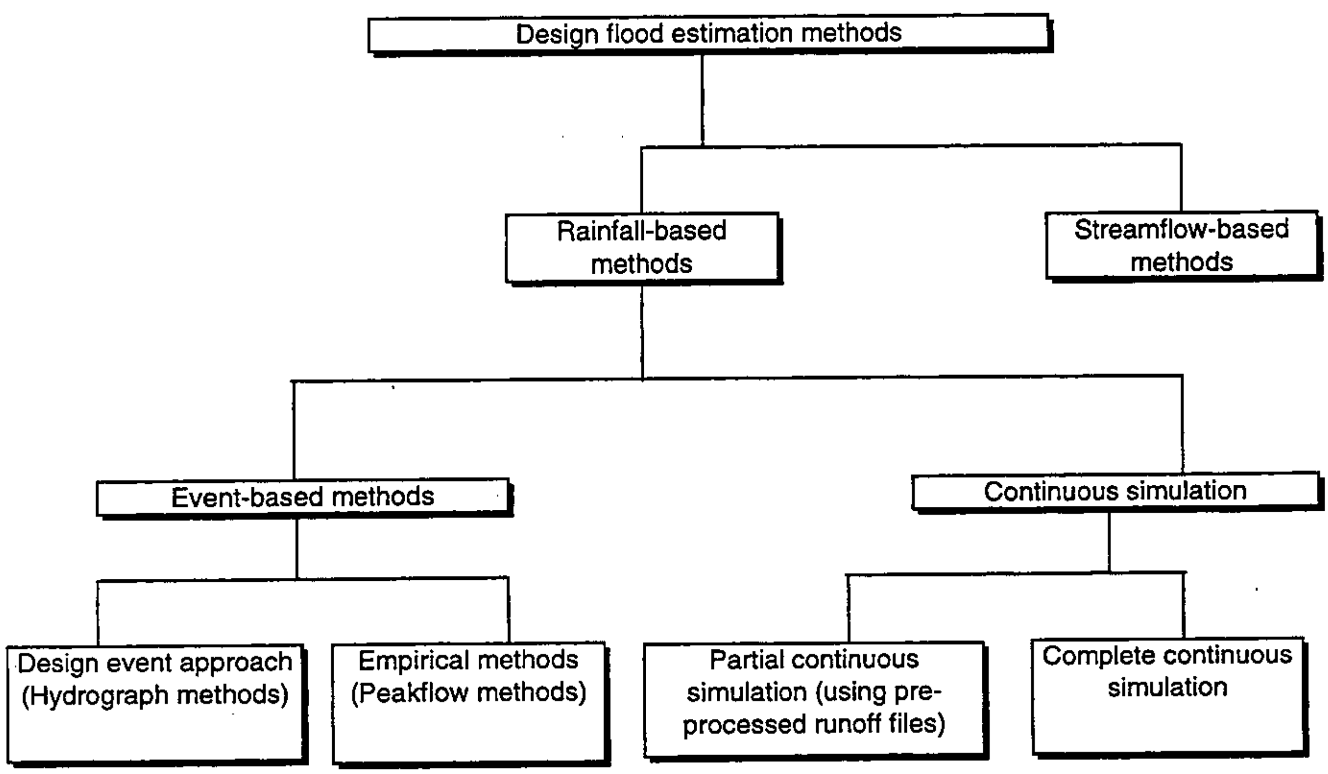

Figure 1.1 Design flood estimation methods

Streamflow-based methods give estimates of floods by analysing observed streamflow data. An example of these is direct flood frequency analysis in which a design flood of a specified probability is estimated by frequency analysis of observed streamflow data at the design location. Its application is limited to situations where a sufficiently long streamflow record is available.

In rainfall-based methods, estimation is based on the analysis of rainfall data (and possibly streamflow data also) and this normally involves use of some rainfall-runoff model. Some important aspects of these methods are: (i) normally rainfall records are longer than streamflow records and use of these rainfall records in conjunction with a rainfall-runoff model allows more accurate flood estimates than those obtained from streamflow records alone particularly at sites with limited streamflow data; (ii) catchment conditions may change with time (e.g. due to land-use change) thus rendering portion of the streamflow data of limited use, whereas climatic conditions remain rather stable over time, and thus long series of rainfall data can be used to obtain more accurate flood estimates; (iii) areal extrapolation of rainfall data can be achieved more easily than that of streamflow data; and (iv) physical features of a catchment are incorporated in a rainfall-runoff model which facilitates extreme flood estimation. These features of rainfall-based methods make these appropriate for catchments with little or no recorded streamflow data, or in situations where catchment conditions have changed significantly over the period of record, and for extreme flood estimation.

Rainfall-based methods can be subdivided into event-based methods and continuous simulation. One example of event-based methods is the current design event approach, the common procedure adopted for obtaining design flood hydrograph from the design event.

Various rainfall-based flood estimation methods are described below.

2.2 RAINFALL-BASED DESIGN FLOOD ESTIMATION METHODS

2.2.1 DESIGN EVENT APPROACH

ARR (I. E. Aust., 1958, 1977, 1987), Beran (1973), Ahern and Weinmann (1982), described the steps involved with the design event approach. The estimation of design flood of a specified annual excedance probability (AEP) by this method is illustrated in Figure 2 and summarised in the following steps:

(i) Select a number of *design storm durations* D1, D2, ... For each of these, obtain a streamflow hydrograph following the steps (ii) to (x), given below.

(ii) Obtain an average rainfall depth from the IFD curve, given the design location, specified AEP and duration.

(iii) Obtain average catchment rainfall using an empirical *areal reduction factor.*

(iv) Select a *rainfall temporal pattern.*

(v) Compute *gross rainfall hyetograph.*

(vi) Select *loss parameters* and compute *rainfall excess hyetograph.*

(vii) Formulate *catchment response model.*

(viii) Select *catchment response parameters.*

(ix) Select *design baseflow.*

(x) Compute *streamflow hydrograph.*

(xi) The rainfall duration giving maximum peak flood is taken as *critical duration*, and the corresponding peak is taken as the *design flood* of the specified AEP.

The key assumption involved in this approach is that the representative design values of the inputs/parameters at different steps can be defined in such a way that they are "AEP neutral" i.e. they result in a flood output that has same AEP as rainfall input. The success of this approach is crucially dependent on how well this assumption is satisfied.

There are no definite guidelines on how to select the appropriate values of the inputs/parameters in the above steps that are likely to convert a rainfall depth of particular AEP to the design flood of the same AEP. There are many methods to determine an input value. A designer is commonly in the situation to select an input value (e.g. median value from a sample of inputs or fitted parameter values) from a wide range. For example, in the case of eastern Queensland, the recommended range of initial loss is 0 to 140 mm (I. E. Aust., 1987). Likewise, other inputs to the design such as critical storm duration, spatial and temporal distributions of the design storm, baseflow values, etc. can also be determined by many methods, the choice of which is totally dependent on various assumptions and preferences of the individual designer. Due to the non-linearity of the transformation process involved, it is generally not possible to know a priori how a representative value for an input should be selected to preserve the AEP.

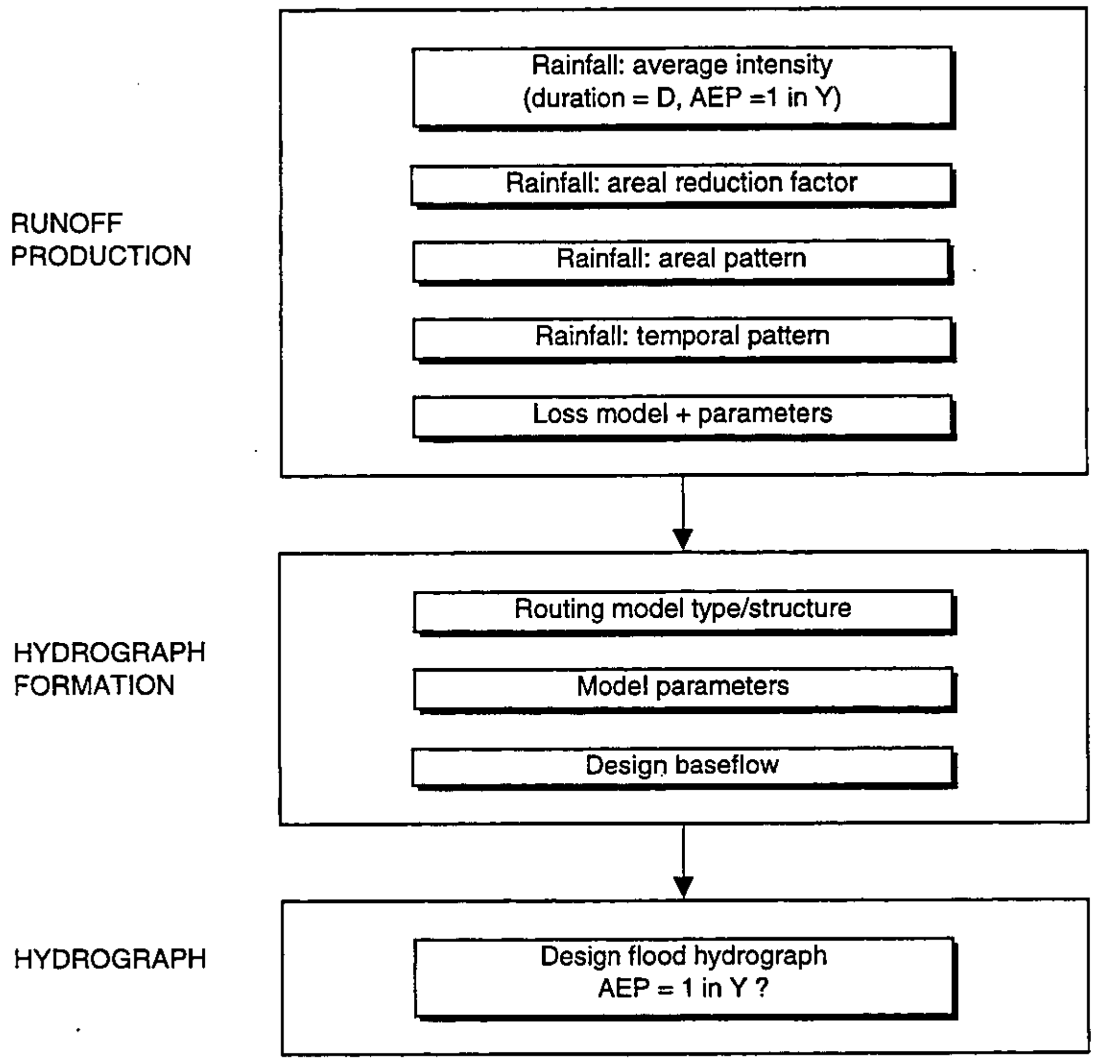

Figure 1.2 Flood estimation by design event approach

For example, critical storm duration is an important factor in converting rainfall input to a flood output of design AEP. The critical storm duration of a catchment depends on the combination of storm factors, loss factors and catchment characteristics, and selection of inappropriate value for any of these factors and inappropriate combination of these factors will result in different critical durations and consequently the AEP of the rainfall not being preserved. An example of inappropriate combination could be indicated by an excessively long critical storm duration for a given catchment size, as found in a number of studies (e.g. Walsh et al., 1991; Hill and Mein, 1996).

The uncertainties in input values to design can be illustrated by a tree diagram (Figure 3) representing a practical design situation where unknown inputs are shown by ranges of values. For example, the storm duration may be D1, D2, D3, etc., or storm losses may take on any value L1, L2, L3, etc. Thus, there are various ways in which a design rainfall and other inputs can be combined to produce a resulting flood. Because of the uncertainty about the correct value of an input to be used in a design situation, except for the rainfall depth for a given duration which is described by a probability distribution, designers tend to adopt median or representative values for those inputs, with the hope that this will lead to a flood estimate of the same probability as that of the design rainfall (Ahern and Weinmann, 1982; I. E. Aust., 1987; Overney, et al., 1995).

In summary, the current design event approach considers the probabilistic nature of rainfall depth but ignores the probabilistic behaviour of other inputs/parameters such as rainfall duration, losses, baseflow. The assumption regarding the probability of the flood output i.e. that a particular AEP rainfall depth will produce a flood of the same AEP, is unreasonable in many cases. This is because the design estimate is sensitive to legitimate but subjective variations in design assumptions (Beran, 1973; NERC, 1975; Russell, et al., 1979; Ahern and Weinmann, 1982). The arbitrary treatment of the various flood producing variables, as done in the current design event approach, is likely to lead to inconsistencies and significant bias in flood estimates for a given AEP.

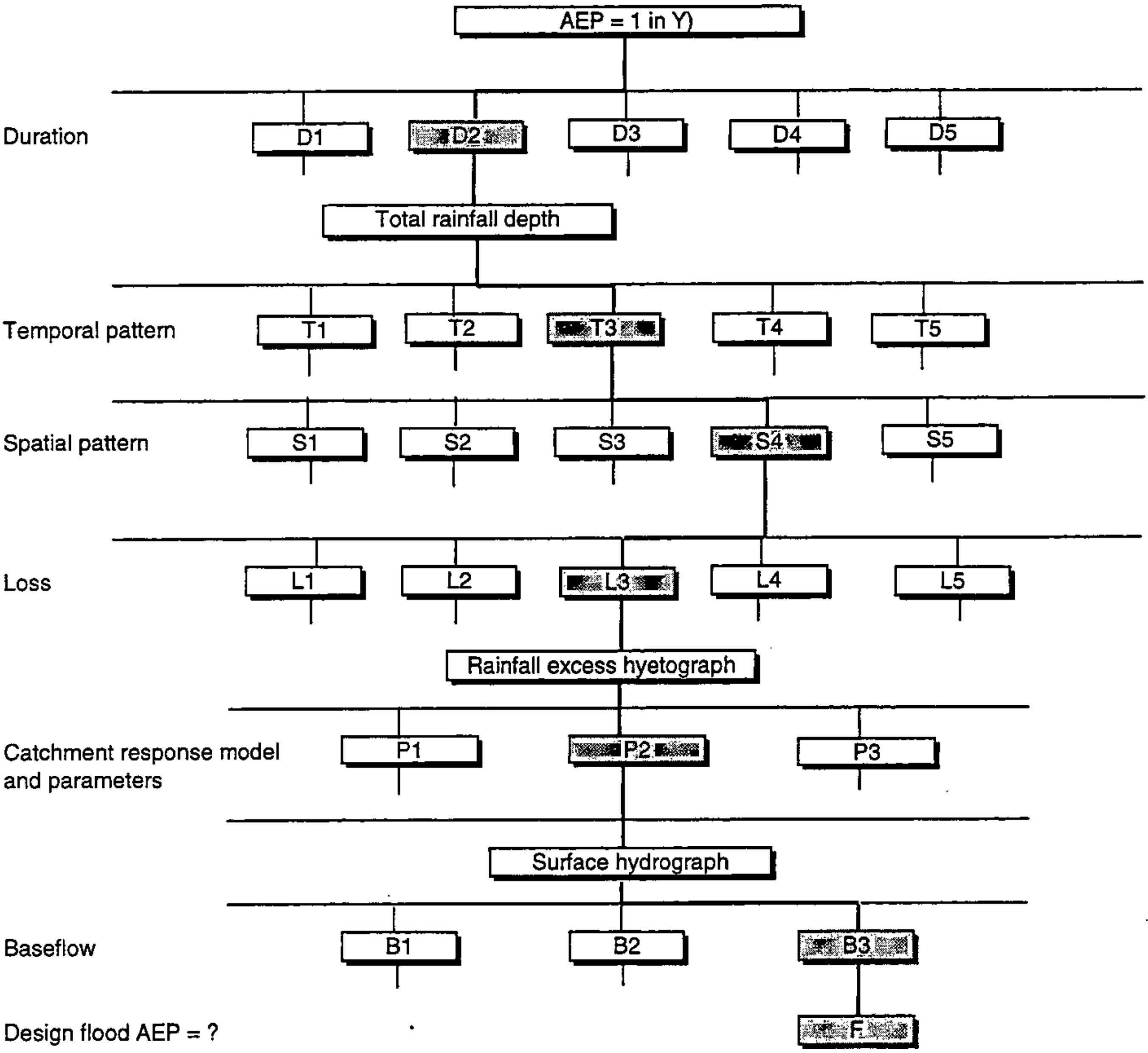

Figure 1.3 Attributes of the design event approach (after Beran, 1973)

2.2.2 ALTERNATIVE METHODS TO THE DESIGN EVENT APPROACH

Several methods have been proposed to overcome the limitations associated with the design event approach: (i) empirical methods (ii) 'improved' design event approach (iii) joint probability approach (iv) continuous simulation (v) runoff files approach. These methods are discussed below.

Empirical methods:

Empirical methods use observed flows to derive one or several coefficients to be incorporated in an equation representing the rainfall-runoff model (James and Robinson, 1986). The most common examples of these are the Probabilistic Rational method (I. E. Aust., 1987) and USGS quantile regression method. These methods are of "black box" model type i.e. they do not incorporate any hydrologic knowledge in the system but are simply a means of converting a known rainfall input into a design flood output.

In principle, empirical methods can overcome the basic limitations of the design event approach because of the following characteristics (James and Robinson, 1986; I. E. Aust., 1987):

- by comparing values of the same probability obtained from frequency analysis of observed floods and rainfalls, a flood of a selected probability is directly linked with rainfall of the same probability; and
- in doing so, effects of other variables affecting floods are automatically considered (they are part of the black-box).

However, the scope of the empirical methods for practical flood frequency applications is limited because they share the principal limitation of black-box methods: their direct application is restricted to within the range of conditions they have been calibrated to and thus should be extrapolated only with extreme caution. As an example, the runoff coefficients used with the Probabilistic Rational Method in ARR (I. E. Aust., 1987) have been determined for a limited range of catchment sizes and characteristics; the extrapolation of these coefficients to ungauged catchments of different sizes and catchment conditions is therefore questionable. The empirical methods also have a limited range of application as they produce only peakflow estimates.

'Improved' design event approach:

As discussed in Section 2.2.1, limitations of the current design event approach come partly from the uncertainties involved in selecting input and parameter values in design. Thus, an obvious but simple method to improve design flood estimates is to use the same flood estimation procedure but with better estimates of parameters and inputs to the design.

A number of research projects have been carried out along this line. Haan and Schulze (1987) characterised the uncertain behaviour of maximum water abstraction S to be used in a simple Soil Conservation Service (SCS) curve number equation (Soil Conservation Service, 1972) for peak flow estimates Q by a probability distribution. Different values of S were substituted into the equation to find corresponding values of Q. Later on, a rigorous way to analyse parameter uncertainties using the Bayesian theorem and a Monte Carlo method for deriving flood frequency curves using the SCS unit hydrograph (Soil Conservation Service, 1972) was proposed by Edwards and Haan (1989). Application of the Bayesian theorem allows incorporation of new (or experimental) information with previous (or prior) probability assessments to yield new (or posterior) probabilities of events of interests (Haan, 1977). Overney et al. (1995) calibrated optimal parameter values for a unit hydrograph model by applying an optimisation technique. Using the SCS abstraction method to simulate runoff, curve number values were described by a probability distribution obtained from observed data. Monte Carlo simulation was then used to generate flows from combinations of the generated stochastic rainfall, unit hydrograph model parameters and curve number values.

Although the above methods recommend different ways to allow for uncertainty in estimates of individual parameters in the design process, these are subject to a common basic limitation of the current design event approach. That is, probability of the resulting flood is still assumed to be equal to that of the causative rainfall.

Joint probability approach:

The joint probability approach recognises that any design flood characteristics (e.g. peak flow) could result from a variety of combinations of flood producing factors, rather than from a single combination, as in the design event approach. For example, the same peak flood could result from a small storm on a saturated basin or a large storm on a dry basin. Thus, it appears that a joint probability approach which considers the outcomes of events with all possible combinations of input values and, if necessary, their correlation structure, will lead to better estimates of design flows (Ahern and Weinmann, 1982).

ARR (I. E. Aust., 1987) recognised the theoretical superiority of the joint probability approach. It mentioned that the stochastic nature of the runoff producing variables can be incorporated into the flood estimate by means of transition probability matrices, or a large number of simulations using values drawn randomly from assumed probability distributions of the variables.

By using the same component models as the current design event approach but treating inputs and parameter values to the design as random variables, the joint probability approach obviously attempts to eliminate subjective criteria in specifying input values. The flood output, consequently, will also have a probability distribution instead of a single value. Therefore the method is theoretically superior to the design event approach and regarded as an attractive design method (I. E. Aust., 1987).

Continuous simulation:

Another alternative to the design event approach is continuous simulation using deterministic catchment models or rainfall-runoff process models (I. E. Aust., 1987). Examples of this approach can be found in Crawford and Linsley (1966); Linsley and Crawford (1974); James and Robinson, (1986); Huber et al. (1986); and Linsley et al. (1988).

Generally speaking, continuous simulation models aim to represent the major processes responsible for converting the catchment rainfall inputs into flood outputs. They generate outflow hydrographs over long periods of time from input of historical rainfall series, potential evaporation and possibly other climatological data. One important characteristic of these models is the continuous use of a water budget model for the catchment so that conditions antecedent to each storm event are known. Time steps used in these models (for design flood estimation) are usually from one hour to one day, sometimes maybe as short as 5 or 15 minutes, and the simulation period is often up to hundreds of years.

Continuous simulation is regarded as having potential for solving the limitations of the current design event approach for a number of reasons:

- it eliminates the need for using synthetic storms by using actual storm records (Russell, 1977);
- it eliminates subjectivity in selecting antecedent conditions for the land surface since a water budget is accounted for in each time step of the simulation and thus automatically logs antecedent moisture conditions (James and Robinson, 1986);
- it handles antecedent conditions correctly because the continuous time series of flows includes all effects of antecedent conditions (Huber, et al., 1986);
- it overcomes the problem of critical storm duration because it simulates the resultant flows for all storms (Lumb and James, 1976); and
- it undertakes a frequency analysis of the variable of interest (peakflow rates, flow volumes, pollutant washoff, etc.) by statistically analysing the time series of model

outputs, as opposed to assuming equal probability of floods and causative rainfalls (Huber, et al., 1986).

The main problems with the approach arise from the difficulties in adequately modelling the soil moisture balance, synthesising long records of rainfall and evaporation at the appropriate temporal and spatial resolution, and accounting for correlations between inputs.

Other difficulties associated with the application of continuous simulation as described by Lumb and James (1976), Ahern and Weinmann (1982), James and Robinson (1982), and ARR (I. E. Aust., 1987) are:

- loss of sharp events if relatively long time steps are used;
- significant time and effort required in gathering the precipitation and other climatologic data needed for simulation of long continuous sequences (extensive data requirements);
- management of large amount of time series output (data management); and
- expertise required to determine parameter values which best reproduce historical hydrographs (model calibration effort).

Despite the above limitations, continuous simulation may prove to be the most powerful means of estimating flood frequencies from rainfall in the near future.

Runoff files approach:

The runoff files approach is a modified version of continuous simulation, aiming at providing the advantages of continuous simulation without major data collection and calibration efforts for different model applications (Lumb and James, 1976).

Like continuous simulation, the runoff files approach requires a long record of historical storms and a rainfall-runoff model. From these, it calculates a time series of unit area unrouted runoff for a range of different land surfaces, each characterised by land use, vegetal cover, soil types, land slopes, etc. These runoff volumes are then stored on separate computer files (called runoff files), each of which represents one type of land surface. For a particular catchment to be modelled, a runoff file of similar land surface condition is selected. The selected runoff file is finally combined with a routing model to produce time series of flood hydrographs, flow volumes or any other flood characteristics of interest.

The runoff files approach has been applied by Sode (1972), Lumb and James (1976), Russell (1977), and King County (Washington, 1995). The major advantage of the runoff files approach compared with continuous simulation modelling is the reduced need for expertise at the application stage, and the exclusion of data and cost of repeated model calibration for individual watersheds. Nevertheless, the method is mainly useful to urban catchments where repeated hydrologic evaluation of options for land-use control, development of detention storage, channel modifications, etc. is required.

Method (s) having greatest potential:

The alternatives to the current design event approach include empirical methods, the 'improved' design event approach, the joint probability approach, continuous simulation and the runoff files approach. Of these, only continuous simulation (possibly also in its simplified form as the runoff files approach) and the joint probability approach have the potential of fully solving the limitations of the current design procedure. The two approaches are similar in their (deterministic) modelling of the hydrograph formation process (runoff routing) but differ in the form of their basic inputs and how they use the inputs to represent the runoff generation phase.

Continuous simulation is regarded as a promising method because it eliminates subjective selection of antecedent moisture, critical storm duration and assignment of equal probability to runoff. It requires continuous modelling of the dominant processes involved in runoff production, i.e. evapotranspiration, soil moisture redistribution, groundwater flow and interflow. Consequently, in comparison with the joint probability approach, it requires fewer inputs that have the potential to introduce bias and uncertainty, but involves a much more difficult modelling process, with its attendant sources of uncertainty.

The joint probability approach calculates the probability of an output by considering all possible combinations of design inputs, each input being treated as a random variable. Application of the method thus requires determination of probability distributions of flood producing inputs and combining them with an appropriate rainfall-runoff model to produce a probability distribution of the flood output. The joint probability approach is more closely related to current design practice and could make use of a large body of existing experience and design data. Appropriate probability frameworks to implement this approach are also readily available. It thus has the potential to become a useful and improved design flood estimation tool in the very near future with relatively modest efforts.

3. JOINT PROBABILITY APPROACH TO DESIGN FLOOD ESTIMATION

The joint probability approach considers the inputs/parameter values to the design as random variables, and thus attempts to eliminate subjectivity in selecting input values. In this approach, flood output has a probability distribution in stead of a single value. The method of combining probability distributed inputs to form a probability distributed output is known as the derived distribution approach. The procedure of determining a derived flood frequency distribution for a catchment can be thought of as a combination of deterministic and stochastic hydrologic modelling (Laurenson, 1974; Laurenson and Pearse, 1991). The stochastic elements are reflected in the adopted distributions of the input variables and parameters, as well as in the assumed correlation structure. These are generally determined not only from the data at the site but from a broader information base for the region (Weinmann, 1994). The transformation of catchment inputs into outputs is deterministic in nature, and is achieved by means of a rainfall-runoff model.

The statistical basis of the joint probability distribution and derived distribution approach is presented in Appendix A.

A derived probability distribution can be found in two ways: (i) analytical methods and (ii) approximate methods. The choice of a method to compute a derived distribution from these options is influenced mainly by the level of analytical skills and the computer resources available for the task (Weinmann, 1994).

The methods of determining derived distributions adopted previously are discussed below (Section 3.1). The flood producing variables that are important in a joint probability modelling framework are discussed next (Sections 3.2 and 3.3). The correlation between the flood producing variables is examined in Section 3.4

3.1 DIFFERENT METHODS

3.1.1 ANALYTICAL METHODS

There are many examples where an analytical approach has been used for deriving flood frequency distributions. Bates (1994) and Sivapalan et al. (1996) presented a summary of these studies. A review of the previous work is presented below with a particular focus on the results of the studies in relation to practical applicability of the methods. The studies are grouped here depending on the runoff production/runoff routing method adopted.

A. Methods based on Eagleson's kinematic runoff model

The derived flood frequency approach was pioneered by Eagleson (1972) who derived the probability distribution of peak streamflow from a given catchment from the probability distributions for climatic and catchment characteristics by using a kinematic model for runoff from an idealised V-shaped flow plane. This approach assumed that storm characteristics (duration and intensity) are independent random variables with a joint exponential probability density function. He used the empirical areal reduction factors to convert point rainfall to catchment-average rainfall. His rainfall-runoff model utilised a partial area runoff generation model with an infiltration capacity that was assumed to be constant during an event, as well as across all events. The runoff routing model utilised kinematic wave equations for both overland flow and channel flow. The method has only limited practical applicability for reasons such as (i) the assumption of independence between rainfall duration and intensity is not likely to be satisfied; (ii) the analytical kinematic wave formulation is applicable to simple V-shaped geometry; (iii) the number of parameters of the derived distributions is large (Wood and Hebson, 1986) and some (e.g. overland flow surface parameter, stream bed parameter) are difficult to obtain for a particular catchment; (iv) the lumped roughness/slope parameters used have no direct physical interpretation; and (v) the mathematical difficulties associated with the flow equations preclude analytical derivations for more complex catchments (Hebson and Wood, 1982).

Wood (1976) extended the method of Eagleson (1972) by allowing the infiltration capacity (which represented random antecedent wetness) to vary between storm events according to some assumed distribution. He found that the random variability in the antecedent wetness of the catchment could have a substantial effect on the predicted return periods.

Cadavid et al. (1991) applied a derived distribution approach to small urban catchments which included Eagleson's (1972) rainfall model, Philip's (1957) infiltration equation, and the kinematic wave model for runoff routing. Their model did not show good fits, particularly for low AEP floods. It was noted that Eagleson's (1972) exponential rainfall model may not be a satisfactory representation of rainfall processes that cause floods and that the estimation of the rainfall model parameters may have a major effect on the success of the derived distribution approach.

Muzik (1994) presented a study illustrating how the physical laws applicable to runoff affect the probability distribution of peak flows. The analytical solution of the kinematic wave equations of overland flow from an impervious runoff plane due to uniformly distributed rainfall was adopted. It was found that the distribution of peak discharge of elemental runoff approaches the parent distribution of rainfall intensity when the physical parameters of the runoff plane exceed certain critical values. It was then argued that the distribution of floods on a natural watershed, when runoff conditions are maximised, should similarly approach a limiting distribution in its upper tail governed by probability distributions of rainfall parameters in the region.

B. Methods based on geomorphologic unit hydrograph

Hebson and Wood (1982) and Diaz-Granados et al. (1984), in application of derived flood frequency distributions, adopted Eagleson's (1972) rainfall model and runoff routing models based on the geomorphologic unit hydrograph (GUH) concept (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980; Rodriguez-Iturbe et al., 1982a, b). The GUH is an instantaneous unit hydrograph derived from considerations of drainage network structure.

Hebson and Wood (1982) used Eagleson's (1972) partial area runoff production model and their runoff routing model was based on the third-order GUH of Rodriguez-Iturbe and Valdes (1979). They attempted to incorporate the effects of catchment scale and shape into the runoff dynamics, and suggested that the GUH would be more suitable than Eagleson's (1972) kinematic wave method to derive the joint probability distribution of floods. Their procedure was tested on two Appalachian Mountain catchments and the results compared well with the observed streamflow data.

Diaz-Granados et al. (1984) adopted an infiltration excess runoff generation model (covering the whole catchment) based on Phillip's (1957) representation of the infiltration process. Their runoff routing model was based on a later development of the GUH theory (Rodriguez-Iturbe et al., 1982a). They tested their procedure against the sample flood frequency distributions for arid and wet climates and achieved good and reasonable fits, respectively.

Wood and Hebson (1986) extended the work of Hebson and Wood (1982) to the study of flood frequency similarity between catchments. The model involved use of dimensionless rainfall inputs and a dimensionless basin response function. The probability distribution of the dimensionless areal rainfall was derived assuming a gamma distribution that incorporated basin size and rainfall areal correlation structure. This is different from the use of empirically derived areal reduction factor as used by Eagleson (1972). Wood and Hebson (1986) adopted the scaling of rainfall duration by a characteristic basin time, a function of basin size. The same scaling factor was used to scale the GUH of catchment response. In deriving the joint probability distribution, they assumed a uniform rainfall intensity over the excess storm duration and independence between average areal storm depth and excess storm duration. The derived dimensionless flood frequency distribution was a function of four parameters: a geoclimatic rainfall scaling factor, Horton's length ratio, the average storm duration and a characteristic basin response time. The method involved evaluation of complex form of integration for which semi-analytical or numerical procedures have been suggested. The method devoted more attention to the problem of flood frequency similarity among river basins rather than to the estimation of flood peaks.

Moughamian et al. (1987) examined the performance of the derived flood frequency models of Hebson & Wood (1982) and Diaz-Granados et al. (1984). They applied these methods on three catchments, and found that both models performed poorly in every catchment when compared to sample distributions. The significant errors in the derived flood frequency curves resulted from the cumulative effect of a large number of relatively small errors in the rainfall inputs and rainfall-runoff models. This study suggested that fundamental improvements are needed before they can be applied with any confidence (Sivapalan et al., 1990).

Sivapalan et al. (1990) described a derived flood frequency model using a partial area runoff generation model, based on an extension of TOPMODEL to include Hortonian runoff generation estimated by the Philip's (1957) infiltration equation, and a runoff routing model based on the generalised geomorphologic unit hydrograph (GUH) and consistent with partial area runoff generation. This work was an extension of the previous work by Hebson and Wood (1982), Diaz-Granados et al. (1984) and Wood and Hebson (1986). The areal rainfall intensities were sampled from a gamma distribution that accounted for the effects of areal averaging similar to the geoclimatic scaling factor of Wood and Hebson (1986). The method allowed for the variability of antecedent moisture conditions between storms and the effects of catchment scale both on the rainfall input distributions and in runoff generation. The work was mainly devoted to provide a greater understanding of the interrelationships that underlie the storm response of catchments of different scales and physical characteristics. The results were presented in a dimensionless framework to study hydrologic similarity of catchments. The approach placed more importance on the 'production phase' rather than the 'transfer phase' of the rainfall-runoff process, and mainly dealt with 'scaling and similarity issues' rather than flood estimation. The developed flood frequency model was not tested on actual catchments.

Troch et al. (1994) modified the procedure of Sivapalan et al. (1990) in an application to a catchment in Pennsylvania. The main difference was the use of a width function-based runoff routing scheme by the former. The application was limited to the investigation of the sensitivity of flood frequencies to a number of model parameters. The practical applicability of the method to the design flood estimation problem was not demonstrated.

C. Methods based on U. S. Soil Conservation Service's curve number procedure

Using a simple equation from the U. S. Soil Conservation Service (SCS) curve number method (Soil Conservation Service, 1972) to estimate runoff from rainfall R and maximum water abstraction S, Haan and Edward (1988) derived the joint probability density function of runoff Q and S, from which the marginal distribution of Q was determined. The equation derived is strictly applicable to the SCS curve number method, and the derivation procedure becomes much more difficult in situations where a more complex transformation between rainfall and runoff is required.

Raines and Valdes (1993) modified Diaz-Granados et al.'s (1984) approach where the SCS curve number procedure was used instead of Philip's (1957) infiltration equation to estimate runoff. The new model and those of Hebson & Wood (1982) and Diaz-Granados et al.'s (1984) were applied to four catchments in Texas, and it was found that none of the models was able to fit satisfactorily the observed flood frequency curves. They noted that rainfall model parameters were the major source of error.

Becciu et al. (1993) adopted a derived distribution technique in flood estimation for ungauged catchments. A two component extreme value distribution was adopted to derive the regional growth function and a derived distribution approach was adopted to estimate the index flood. In the derived distribution the point rainfall was described by a Poisson distribution; intensity and duration of rainfall were assumed to be mutually independent random variables with exponential distributions and the spatial reduction of precipitation over the basin area was accounted for by means of an areal reduction factor. The approach used a curve number method and linear reservoir cascade theory to model surface runoff. The application of the methodology to catchments in Northern Italy showed its capability to satisfactorily reproduce the frequency distribution of the observed data.

D. Methods based on other types of rainfall-runoff models

Beven (1986) adopted a joint probability approach to flood estimation where he used rainfall distributions similar to Eagleson (1972), a rainfall-runoff model that combined the topographically-based TOPMODEL with a routing model based on the catchment's width function. In this study, Beven also investigated the change of processes with increasing severity of floods, e.g. the increase of the proportion of saturated area with decreasing AEP.

Haan and Wilson (1987) presented a methodology for computing runoff frequencies from the probabilistic behaviour of rainfall and other factors affecting runoff. They considered two runoff variables: runoff volumes and peak flows. The derived distribution of peak flows was based on the Rational method:

$$Q = CIA \tag{3.1}$$

They noted that runoff coefficient (C) reflects many hydrologic factors including antecedent conditions. If C is calculated from Equation 3.1 from several observed storm events, C is found to be a random variable reflecting otherwise unquantified variations in the hydrologic conditions of the drainage area. The probability distributions of C and I were described by Beta and Extreme Value Type I distributions respectively. They used numerical integration to obtain derived distribution under the assumption of independence of C and I. They found that consideration of runoff coefficients as a random variable provided larger peak flows than that obtained assuming C as a constant, particularly at higher returns periods. This was in agreement with the earlier observation of Schaake et. al. (1967) that C may be larger for storms with greater return periods. This has also been recognised in

ARR (I. E. Aust., 1987). Haan and Wilson (1987) demonstrated the appropriateness of the joint probability approach but did not make any clear recommendation of the use of this approach, and suggested further study before 'any sweeping conclusion can be made'.

Sivapalan et al. (1996) utilised the derived flood frequency methodology to investigate the link between process controls and flood frequency. Their attention was particularly restricted to the processes which contribute to the shape of the flood frequency curve. They used intensity-frequency-duration (IFD) curves for the probabilistic description of rainfall inputs. A three parameter Weibull distribution was fitted to the observed storm durations which gave an estimate of the marginal distribution of duration. They specified the joint distribution of rainfall intensity and duration by multiplying IFD curves (conditional distributions) with marginal distributions of duration. They utilised an event-based rainfall-runoff model based on linear reservoirs under a quasi-analytical modelling framework. They identified that temporal patterns, multiple storms and the nonlinear dependence of runoff coefficients on event rainfall depth are the major factors controlling the shape of the flood frequency curve. They introduced the use of IFD curves in the derived distribution procedure that would help to unify theoretical research on derived flood frequency with traditional design practice, largely based on the use of the IFD curves.

Bloschl and Sivapalan (1997) investigated the effects of various flood producing factors (runoff coefficients, antecedent conditions, storm durations and temporal pattern) on flood frequency curve in a derived distribution frame work. Like Sivapalan et al. (1996) they used IFD curves for the rainfall model. They found that non-linear runoff generation (reflected in increasing runoff coefficient with event size), random antecedent soil moisture (reflected in random runoff coefficients), and non-linear routing (reflected in faster runoff response with event size) all translate into steeper flood frequency curves than in the linear case. They argued that this non-linearity may be the reason that flood frequency curves tend to be much steeper than rainfall frequency curves. It might be noted that the different slopes and shapes of rainfall and flood frequency curves have been observed for many catchments. It could be argued from the observation of Bloschl and Sivapalan (1997) that these differences are an expression of significant non-linearity in runoff generation and hydrograph formation processes.

They found storm temporal pattern to be of critical importance for the flood frequency behaviour and the assumption of equal return periods of the input rainfall depth and output flood associated with the current design event approach "is always grossly in error", a finding which has very important implication in design practice. For the two study catchments in Austria, the design event approach underestimated flood return periods by a factor of at least two but "this factor may be as large as ten".

3.1.2 APPROXIMATE METHODS

Approximate methods are often used in hydrology to determine derived frequency distribution. There are two categories of approximate methods: (a) discrete methods – here the continuous distributions of hydrologic variables are discritized and the Theorem of Total Probability is generally used to obtain the derived distribution; and (b) simulation techniques – here random samples are drawn from continuous distributions of input variables, and the resulting outputs are used to determine derived frequency distribution. These methods are described below:

Discrete methods:

One characteristic of these methods is the use of discrete probability distributions to describe hydrologic variables (such as flood peaks, antecedent precipitation index, soil moisture deficit, etc.) even though they are really continuous ones. In practice, this is frequently done by dividing the possible range of a random variable into class intervals. The discrete distribution describing the variable is then represented by either class intervals or a single value (often at the mid-point) for each class interval. The accuracy of the approach depends on the degree of discretization. This method is adopted by many researchers e.g. Beran (1973), Laurenson (1974), Russell et al. (1979), Fontaine and Potter (1993).

One relatively common trend in applying approximate methods to determine flood probability distributions is to use the Theorem of Total Probability to calculate flood probabilities. Examples of this include Laurenson (1974), Russell et al. (1979), Fontaine and Potter (1993). The simplest application of this is made by Fontaine and Potter (1993), in which Equation A8 (Appendix A) is used to calculate flood probabilities. For a given flood, its exceedance probability is the sum of three terms, each being the joint probability of extreme rainfall and antecedent soil moisture. The latter, modelled by the SCS curve number method (Soil Conservation Service, 1972), is assumed to be represented by three curve numbers. In fact, this over-simplified assumption is one basic limitation of the proposed joint probability approach.

The same concept is applied by Russell et al. (1979) to a rainfall-runoff model represented by three parameters (time of concentration T, infiltration rate I and storage constant R). By using the 60 largest recorded storms and 120 combinations of T, I, R, resulting unit peakflows are calculated. Next, these are plotted on a Gumbel probability paper, from which conditional probabilities of 50 specified peakflows can be determined. These probabilities are then stored in a large matrix with 50 rows (one for each specified peakflow) and 120 columns (one for each combination of T, I, R). Thus, probabilities of the stored flows can be calculated for any basin using Equation A10 (Appendix A) if probabilities of the parameters T, I, R - assumed to be independent - are given. Thus, this is essentially the runoff files approach described in Section 2.2.2 but a joint probability method is employed to calculate flood probabilities. Russell et al. (1979) used actual storm rainfall records instead of a synthetic storm. The Clark rainfall runoff model (Clark, 1945) which provides the basis for the HEC1 model was used in which rainfall is lagged by a time-area curve and routed through linear storage. It was assumed that infiltration rate would be constant for any particular storm.

The most general application of the Theorem of Total Probability is described by the 'transformation matrix' approach, developed by Laurenson (1974). The method requires division of a design problem into a sequence of steps, "each step transforms an input distribution into an output distribution, which becomes the input to the next step" (Laurenson, 1974). The transformation represents the deterministic relation between input and output of a step. In applying the method, input, transformation relation and output should be expressed in matrix form. One particular value of the transformation matrix represents the conditional probability of obtaining an output value given a value of input. The probability of an output value is calculated by summing, for all possible input values, the joint probabilities of the output with those inputs, expressed by Equation A8. The 'transformation matrix' method provides a wide range of applications (Laurenson, 1973; Laurenson, 1974; Ahern and Weinmann, 1982; Laurenson and Pearse, 1991) when the stochastic nature of the hydrologic system needs to be accounted for. For example, Laurenson (1973) and Ahern and Weinmann (1982) used a transformation matrix to link the frequency distribution of peak inflows and storage contents to calculate frequency distributions of peak outflows. In design flood estimation, the method has been applied to combine the frequency distributions of rainfalls and losses to obtain a frequency distribution of peakflows (Ahern and Weinmann, 1982) and to calculate the probability distribution of probable maximum precipitation (Laurenson and Pearse, 1991; Pearse and Laurenson, 1997).

The above examples demonstrate how the Theorem of Total Probability can be applied for calculating design flood probabilities. If all the random variables involved in the design are independent, computation of flood probabilities becomes very simple once probabilities of those input variables are given. For the case of dependent variables, application of the theorem becomes relatively difficult.

Beran (1973) presented a procedure that sampled the possible ways in which a storm of a given AEP could cause floods, and derived their joint probability distribution. The continuous distributions of variables (rainfall duration, rainfall temporal pattern, catchment wetness index CWI) were discretized into class intervals so that each variable assumed one of a finite number of possible values to which a probability weight was attached. The probability weight for a variable was taken as proportional to the probability of occurrence within the class interval (obtained from the observed data). Rainfall variables were assumed to be independent. This assumption of independence and the discretization allowed considerable simplification in the simulation. The unit hydrograph method was used as catchment response model. In applying the method, smoothing of flood probability distributions may be required as a result of discretizing continuous distributions into class intervals.

The method seems to have the potential of being applied in practice, and hence is illustrated here. Let p_i be the weight of the *i*th duration D_i, q_j be the weight of the temporal pattern (T_j), r_k be the weight of the *k*th CWI C_k and Q_{ijk} be the flood magnitude resulting from the combination of D_i, T_j and C_k. Assuming independence, the probability to be associated with Q_{ijk} is $W_{ijk} = p_i q_i r_k$.

A number of investigators have used numerical methods to determine derived flood frequency distributions. As reported by Hebson and Wood (1982), Leclerc and Schaake (1972) used a finite difference scheme to solve Eagleson's (1972) equations. Conceptually, this numerical approach can be used for the more complex natural catchments that are mathematically intractable from an analytic point of view (Hebson and Wood, 1982). Shen et al. (1990) presented a study on derived distribution that used numerical integration to determine the derived distribution. They used a Poisson process for arrival of storm events, exponential distributions for rainfall intensity and duration, Phillip's equation for infiltration capacity, and the kinematic wave equation to formulate a rainfall runoff model. They developed relationships between hydrograph characteristics (time to peak, peak flow magnitude, and time of concentration) and rainfall and basin characteristics. For the small basins used in the study, the soil types and initial soil moisture were found to have strong influence on floods of a given frequency. The results of the study are applicable to given ranges of basin characteristics only.

Simulation techniques:

Beran (1973) adopted a simulation technique in that the sampling was conducted across all combinations of storm depth and duration. This generalised simulation produced lower flood values at smaller return periods than the expected flood following storms of that same return period. The important results from Beran's (1973) study are that (i) CWI is the most important variable and a small change in it has a marked effect on the resulting flood; and (ii) T-year return period storms tend on average to give rise to more floods with return period less than T-years than floods of return period greater than T-years. The Flood Studies Report (NERC, 1975) documented the results of the study by Beran (1973). This report mentioned that a rigorous solution to the problem that the design should correspond to a specific return period of flow requires that every combination of the flood producing variables should be used with the rainfall runoff model and the resulting population of peak flows analysed as if it was an annual maximum series (NERC, 1976, p. 376).

As reported in Sivapalan (1996), Tavakkoli (1985) adopted a simulation approach to derive flood frequency curves for an Austrian catchment in that he considered the dependence of rainfall intensity and duration, multiple events, within-storm time patterns (temporal patterns), and variable runoff coefficients. The method resulted in slight overestimation of flood peaks which he mainly attributed to the runoff generation model.

Muzik and Beersing (1989) studied the transformation process of probability distributions of rainfall intensity into the probability distributions of peak flow for the case of runoff from a uniformly sloping impervious plane. Normal, two-parameter gamma and exponential distributions of rainfall intensity were used in Monte Carlo simulations to obtain densities of peak discharge. To compute the peak discharge, kinematic wave and experimentally derived relations were used. The rainfall-runoff relation of the form $q_p = ai^m$ was considered. Some important conclusions from the study were: (i) Physical factors which contribute to negative skewness are those which cause m to decrease. In natural basins, a sudden increase in a basin storage capacity due to overbank flow, storage in flat and swampy areas, underground storage etc. may result in a negative skew of the peak flows. (ii) Runoff planes having flat slopes and large hydraulic resistance will generally produce more skewed distributions of peak discharge and lower values of the mean than steeper and hydraulically more efficient planes will, subject to the same rainfall inputs. The study was carried out in laboratory conditions at a basic level of an experimental rainfall runoff process, and the method was not tested on real catchments.

Muzik (1993) presented a physically based stochastic approach to rainfall-runoff modelling in that a modified SCS curve number method was used in Monte Carlo simulation to obtain a derived distribution of peak discharge. From the analysis of rainfall-runoff data from 55 watersheds in Alberta foothills, Canada, a general relationship was established between maximum potential retention S and five day antecedent rainfall (P5). The initial abstraction and five day antecedent rainfall were assumed to be random variables having log-Pearson type III distributions. The steps involved in the simulation are: (i) generation of a random value of P5; (ii) from the relationship between P5 and S obtaining the maximum potential retention S; (iii) generation of a random value of the initial abstraction Ia; (iv) generation of a random value of total rainfall P; and (v) computation of rainfall excess depth.

The rainfall excess depth was then transformed deterministically by means of the unit hydrograph method into a flood hydrograph. The procedure was repeated many times (in the order of thousands) to obtain a large sample for determination of the derived distribution of peak discharge. The method was applied to some catchments in Rocky Mountains (Canada). The important conclusions were that even if the true distribution of rainfall input, maximum potential retention, initial abstraction etc. are not exactly known, effects of their variation within physically reasonable limits can be assessed, and considered in water resources planning and design by using Monte Carlo simulation. He further noted that using stochastic inputs and deterministic modelling, Monte Carlo simulation provides an excellent approach to study flood probabilities.

Durrans (1995) presented a simulation procedure to determine derived flood frequency curve for regulated sites (such as downstream of dams). The procedure consisted of the following steps: (i) random sampling of unregulated annual flood peak and unregulated flood volume; (ii) random sampling of a dimensionless initial reservoir depth and dimensionless gate opening area; (iii) routing the inflow hydrograph through the reservoir; (iv) replication of steps (i) to (iii) N times (N in the order of thousands) to obtain N outflow hydrograph peaks which are used to empirically define the regulated flood frequency distribution. The method has been described as "an integrated deterministic-stochastic approach to flood frequency analysis". It can also be applied to more general situations to determine flood frequency curves other than outflows from reservoirs.

Bloschl and Sivapalan (1997) adopted a Monte Carlo simulation for mapping rainfall return periods to runoff return periods. The simulation consisted of the following steps: (i) draw storm durations from an exponential distribution; (ii) draw precipitation probabilities from a uniform distribution $P[0;1]$ and calculate precipitation return period, T_p , from $T_p = 1/(1-P)/m$ where m is the number of events per year; (iii) get rainfall intensities, p, from the IFD curve using the two previous pieces of information; and (iv) fit temporal pattern to rainfall, apply runoff coefficient to estimate rainfall excess, simulate streamflow hydrograph from the selected runoff routing model, and note the peak. Steps (i) to (iv) were repeated 10,000 times which correspond to 10,000 events. At the end, the flood peaks were ranked which allowed assignment of a return period to each event by using plotting positions: $T_q = n/j/m$ where T_q is the return period of the flood, n is the total number of events, and j is the rank. This simulation technique is relatively simple and appears to be a potential method for practical application.

It appears that the mathematical framework adopted by several previous studies can provide some useful guidance in the formulation of a research methodology that is likely to lead to a practical design flood estimation technique based on the joint probability approach. These studies are those of Beran (1973), Russell et al. (1979), Muzik (1993), Bloschl and Sivapalan (1997). These studies are compared in Table 3.1. The methods of Beran (1973) and Russell et al. (1979) adopted discretization of continuous variables into class intervals and assumed independence of variables. The methods of Muzik and Bloschl and Sivapalan (1996) are examples of Monte Carlo simulation.

Table 3.1 Comparison of the studies of Beran (1973), Russell et al. (1979), Muzik (1993) and Bloschl and Sivapalan (1997)

Study	Random variables	Runoff production and runoff routing models	Method of obtaining derived distribution	Salient features of the method	Important results	Comments
Beran (1973)	Storm properties (depth, duration and temporal pattern) and catchment wetness index (CWI).	An empirical equation based on CWI, total rainfall, soil type and landuse for runoff production. Unit hydrograph method for runoff routing.	Simulation.	(i) Continuous distributions of variables were discretized: 12 values of duration, 36 temporal patterns and 12 CWIs were used. A probability weight was assigned to each of the discretized values determined according to its probability of occurrence. (ii) No dependence between variables was considered. (iii) Probability distributions of storm durations and temporal patterns were based on complete storms and obtained from the observed data but existing IFD curves based on storm bursts were adopted for rainfall depth.	(i) The loss parameter was found to be the most influential variable. (ii) The derived flood frequency curves were much flatter than the observed ones.	(i) The method is not a fully generalised simulation approach, it is a combination of the approximate method and the simulation technique. (ii) Definition of storm for obtaining probability distribution of storm durations is not consistent with that of IFD information for rainfall depth.
Russell et al. (1979)	Three model parameters of the Clark runoff model: time of concentration (T), storage constant (R) for the assumed linear reservoir, infiltration function (I). The probabilistic nature of storm properties (duration, depth and temporal pattern) was accounted for through the observed variability of these variables from sixty largest complete storms from fifteen years record.	Clark runoff model (basis of HEC1 model) that included both runoff production and routing.	Approximate method (similar to runoff files approach but coupled with joint probability method).	(i) Complete storms were used. Sixty largest storms from fifteen years record were considered. (ii) Continuous distributions of variables were discretized: 7 values of T, 3 values of I and 9 values of R were used. (iii) Independence of the variables was assumed, however, some combinations were excluded as they appeared to be unrealistic. (iv) Probability distributions of T, I and R were computed from the observed data. (v) Infiltration rate was assumed to be constant during any one storm. (vi) Theorem of Total Probability was used to calculate the AEP of a particular flood value.	(i) The developed procedure can consider stochastic nature of storm precipitation and parameters characterising the basin response. (ii) The results were not adversely affected by the constant infiltration rate.	(i) Method of computing probability distributions of the variables appears to be awkward. (ii) The method was based on very small catchments (up to only 19 km^2). The validity of the method to large catchments need to be tested before applying the method to large catchments. (iii) The method may be treated as a limited simulation approach where sixty largest storms from the observed record generate streamflow hydrographs for 120 different catchment conditions represented by three model parameters. (iv) The method is a good example of using the Theorem of Total Probability in the joint probability approach.

Table 3.1
(contd …)

Study	Random variables	Runoff production and runoff routing models	Method of obtaining derived distribution	Salient features of the method	Important results	Comments
Muzik (1993)	Storm properties (depth, duration and temporal pattern), and parameters of a runoff production model: maximum potential retention (S), five-day antecedent precipitation (P5) and initial abstraction (I_a).	Modified USGS curve number method for runoff production and unit hydrograph method as runoff routing model.	Monte Carlo simulation.	(i) Complete storms were used. Total rainfall depth was described by a Gumbel distribution. (ii) P5 was described by a log Pearson type III distribution. (iii) The correlation between S and P5 was considered.	(i) The derived flood frequency distribution had an acceptable fit to the observed data.	(i) The approach combines knowledge of physical processes with the theory of probability in that knowledge of the processes allows to put reasonable limits on the variable values. (ii) The approach is a good example of applying Monte Carlo simulation in derived distribution.
Bloschl and Sivapalan (1997)	Storm properties (depth, duration and temporal pattern), and antecedent moisture condition through the use of runoff coefficient.	The runoff production model used a runoff coefficient (ratio of runoff to rainfall depth). The runoff routing model is conceptualised in the form of three linear reservoirs in parallel.	Monte Carlo simulation.	(i) Small difference was noticed between the IFD curves based on storm bursts and complete storms, and hence the existing IFD curves based on bursts were adopted but the distribution of storm durations was based on complete storms. (ii) Used IFD curves (as conditional distribution) and distribution of storm duration as marginal distribution. This accounts for the dependence between rainfall depth and duration. (iii) The method did not consider dependence between other variables.	(i) The loss parameter and storm temporal pattern were found to be very important variables. (ii) The assumption of the design event approach that the input rainfall and output runoff frequencies are equal was found to be always grossly in error.	(i) The work of Bloschl and Sivapalan (1997) involved use of a semi-analytical approach mainly; the Monte Carlo simulation was used to map rainfall and runoff frequencies which formed a part of the main work. (ii) This is an example where Monte Carlo simulation has been used successfully in the derived distribution.

3.2 IMPORTANT FLOOD PRODUCING VARIABLES

Estimation of design flood by rainfall-based methods consists of two phases: (i) runoff production (including storm rainfall generation); and (ii) hydrograph formation. The runoff production phase is concerned with quantification of rainfall excess from a rainfall event, and the hydrograph formation phase deals with routing the rainfall excess to obtain a streamflow hydrograph. Each of these phases has stochastic as well as deterministic components. In the runoff production phase, the stochastic inputs are rainfall and loss characteristics; the procedure of converting these inputs into rainfall excess is deterministic. In the hydrograph formation phase, the catchment response parameters are stochastic but the model structure is deterministic; if fixed parameters are used the whole phase is deterministic. The modelling of flood response from a catchment is thus a combination of stochastic and deterministic hydrologic processes (Laurenson, 1974).

The major factors affecting runoff production are: rainfall intensity, rainfall duration, areal pattern of rainfall, temporal pattern of rainfall and storm losses. Factors affecting hydrograph formation are catchment response parameters (model type, model structure, and model parameters) and design baseflow. Most of the hydrologic literature has focused on the latter phase, although it has been known for at least 50 years that the important problem in surface hydrology is determining "what to route" not "how to route" i.e. the production phase is the one that requires more emphasis (Sivapalan et al., 1990).

Of the factors (variables) affecting the rainfall runoff process, only rainfall intensity is described by a probability distribution in the conventional methods. Ideally, all the variables should be treated as random variable, but, consideration of a smaller number of variables without sacrificing much accuracy is preferable, due to smaller data requirements and ease of use in practice. The selection of key variables that should be considered as random variables is described below.

3.2.1 RAINFALL VARIABLES

Rainfall depth is the direct input to rainfall-runoff process, and is, undoubtedly, the most important variable. Its probabilistic nature has been considered in all of the previous joint probability studies, and will need to be considered as a random variable in the present study.

Rainfall duration is an input that has also received considerable attention. This is because, together with the rainfall return period, it directly determines the rainfall depth input to design. Since there is no sound basis for determining the correct duration of storm that should be used to estimate a design flood, the critical storm duration may simply be stated by regulation (Huber et al., 1986) or determined by trial and error (I. E. Aust., 1958, 1977, 1987). In either case, a constant value is assigned to the critical duration of the flood producing rainfall. In the joint probability approach, several investigators have treated storm duration as a random variable (e.g. Eagleson, 1972; Beran, 1973). These studies have considered storm duration and intensity as independent which is likely to result in steeper flood frequency curve (Bloschl and Sivapalan, 1997). It appears that both the storm duration and intensity need to be considered as random variables with a method of accounting for their correlation.

Rainfall temporal pattern is regarded as a very important variable because it can have a major effect on the resulting flood (I. E. Aust., 1987). Differences of up to 50% in flood peaks may result from different assumed temporal patterns (Askew, 1975; Milston, 1979; Brown, 1982; Wood and Alvarez, 1982; Cordery et al., 1984). In addition, adopted extreme patterns may cause computed flood peaks to vary up to 2.5 times for individual observed heavy rainfalls (I. E. Aust., 1987). Viessman et al. (1989) also note that shape and peak magnitude of flood hydrographs are significantly affected by patterns of time distribution of storms. Bloschl and Sivapalan (1997) also found that storm temporal pattern is of critical importance for flood frequency behaviour. These studies and that of Beran (1973) indicate that, compared with loss rate, the effect of time distribution of rainfalls on computed design floods is less significant but still important. It, thus, appears that temporal pattern needs to be considered as a random variable.

The areal rainfall pattern may be an important factor particularly for larger catchments. The modelling of the areal variability of rainfall can be considered at different levels of detail. In the reproduction of actual storm events (model calibration), knowledge of the areal pattern of rainfall is mainly required to allow correct estimation of the catchment average rainfall. For design flood estimates, the effects of areal variability of rainfall are considered mainly through the use of areal reduction factors. An areal rainfall pattern needs to be considered where there are systematic trends in catchment rainfall, such as strong orographic effects or "rain shadow" areas. In large catchments with highly variable rainfall patterns, the random variability of rainfall over different parts of the catchment may also need to be modelled. For smaller catchments, the randomness of areal pattern may be of less importance.

The previously mentioned studies on the joint probability approach did not consider areal rainfall pattern as a random variable (except for Hebson and Wood, 1982). For the present study, the modelling of the rainfall areal pattern as a random variable is considered less important because of the following reasons:

- for most catchments, consideration of rainfall areal pattern as a random variable will have a lesser effect on the results than is the case for rainfall duration, intensity and temporal pattern;
- due to limited rainfall data availability on a catchment scale, it is often difficult to derive its probability distribution; and
- use of a distributed rainfall-runoff model (e.g. RORB) with rainfall inputs for sub-areas computed from local rainfall stations can account for areal rainfall variability to some degree.

3.2.2 LOSS VARIABLES

Of all the variables, loss has probably received the highest attention. Hoang (1997) showed that design flood estimates are very sensitive to values of losses adopted in design. She found that for the Gungoandra Creek Catchment in New South Wales, the peak discharge may increase by up to 120% if an initial loss lower than the median value is assumed. The strong influence of loss values on design flood estimates is based on the fact that a given rainfall occurring on a dry watershed produces significantly less runoff than the same rainfall occurring on a wet watershed (Lumb and James, 1976; James and Robinson, 1986; Haan and Schulze, 1987; I. E. Aust., 1987). Beran (1973), in examining sensitivity of the design flood to alterations of the assumed values of variables, states that 'correct choice of loss rate is in consequence most important'. Thus, it may be concluded that in many cases, loss is the most important factor and should be treated as a random variable in estimating design floods.

3.2.3 CATCHMENT RESPONSE PARAMETERS AND BASEFLOW

Treatment of the variables related to runoff routing and baseflow as purely random variables might not be as important as in the case of rainfall and loss variables because:

(i) The variability in model parameter(s) from event to event may be due to model inadequacy (e.g. lack in coping with non-linearity of rainfall-runoff process) and data error rather than physical reasoning.

(ii) Selection of an appropriate runoff routing model and calibration procedure allows determination of a single set of model parameters for a given catchment which can be applied for the catchment with reasonable confidence.

(iii) Baseflow is an additive component in the present problem. The implication that the baseflow is a random variable is that any streamflow hydrograph has a random baseflow part in it. In reality, the variability in baseflow magnitude in a particular season of the year is relatively small compared to surface runoff. A significant part of the variability of baseflow is systematic (seasonal) rather than random. From this consideration, baseflow would usually have a small effect on the derived flood frequency distribution.

(iv) Selection of an appropriate deterministic method of baseflow estimation can provide satisfactory and consistent results.

It might be expected that the incorporation of the probabilistic nature of the rainfall and loss variables would result in significant reduction of bias and uncertainties in design flood estimates. Consideration of runoff routing and baseflow variables as purely random variables appears to be of secondary importance; thus the effects of randomness of these variables on design flood estimates may be examined as a refinement at a later stage.

It can be concluded from the above discussion that the variables that need to be treated as random variables in the joint probability approach of flood estimation are *rainfall intensity*, *rainfall duration*, *losses*, and *rainfall temporal pattern*. The methods of obtaining probability distributions of these variables are discussed in the next section.

3.3 DISTRIBUTIONS OF IMPORTANT FLOOD PRODUCING VARIABLES

3.3.1 PROBABILITY DISTRIBUTIONS OF RAINFALL DURATION AND INTENSITY

Rainfall duration can relate to complete storms or intense bursts of rainfall within complete storms. The intensity-frequency-duration (IFD) curves used in design practice are based on storm bursts rather than complete storms. For storm bursts with a selected duration (e.g. 2 hour bursts), burst duration is not a random variable.

Most of the previous work (e.g. Hebson and Wood, 1982; Diaz-Granados et al., 1984; Beven, 1986; Cadavid et al., 1991) on the derived flood frequency method has essentially followed Eagleson's (1972) approach to the probabilistic description of rainfall inputs, i.e. rainfall intensity and duration as independent random variables belonging to an exponential or similar distribution. There are two basic problems with this approach:

(i) The assumption of independence between rainfall intensity and duration is not satisfied normally; it is well known that average rainfall intensity decreases as duration increases.

(ii) The probability distribution of rainfall depth was based on complete storms. In design practice, intensity-frequency-duration (IFD) curves are normally used for probabilistic description of rainfall depth which are based on intense rainfall bursts. Thus, the probabilistic descriptions used by these studies are not consistent with design practice.

Sivapalan et al. (1996) adopted the findings of Gutknecht (1977) that the difference between IFD curves based on complete storms and intense bursts for Austrian catchments are not "too great" and used existing IFD curves in their derived flood frequency method. The advantages of the IFD curves are that they explicitly incorporate a dependence between rainfall intensity and duration, and that they are widely used in practice. They fitted Gumbel distributions to the IFD curves, expressed as conditional cumulative distributions, for a given duration. The marginal distribution of rainfall duration was based on complete storms. The method of specifying the joint distribution of rainfall intensity and duration involving IFD curves as described in Sivapalan et al. (1996) appears to be a promising method and worth investigating.

3.3.2 PROBABILITY DISTRIBUTION OF RAINFALL TEMPORAL PATTERNS

Many efforts have been made previously to derive design temporal patterns for use with the design rainfall intensity. These have resulted in a large number of methods for describing the time distribution of rainfall depth (Huff, 1967; Pilgrim and Cordery, 1975; Yen and Chow, 1980; Cordery et al., 1984; I. E. Aust., 1987).

Available temporal patterns of rainfalls may be divided into two groups: (a) 'non-probabilistic' temporal distributions, and (b) 'probabilistic' temporal distributions. These are discussed below.

'Non-probabilistic' temporal distributions:

Most hyetograph patterns belong to this group. Some of the most commonly used are the Chicago pattern (Keifer and Chu, 1957) or patterns of average variability (Pilgrim and Cordery, 1975); the latter aim at preserving probability of exceedance of rainfall in the design flood. Another method is recommended by Yen and Chow (1980) who developed triangular hyetographs that preserve total storm depth and location of peak intensity. Unfortunately, all these patterns cannot be used in the joint probability approach because of some of the following limitations:

- simple hyetograph shapes (for example, triangular) are inadequate in reflecting actual time variations of storms;
- there is no probability associated with a particular pattern; and
- they aim at providing a simple flood output of the same probability as the design rainfall input rather than a distribution of output values.

'Probabilistic' temporal distributions:

Hashino (1986) theoretically derived a stochastic storm pattern which preserves stochastic properties of actual storm hyetographs. Using Freund's bivariate exponential density function (Freund, 1961), equations describing two typical design storm patterns (with the peak rainfall intensity either at the centre or at the end) were derived. A basic assumption used in the development is that rainfall intensity decreases monotonically away from the peak. As a result, the method is limited in its application when more general situations are required.

In contrast to the above theoretical methods, Huff (1967) developed time distribution relations of storms from observed data. Recorded storm distribution patterns in east central Illinois were characterised by mass curves and classified into 4 quartile groups depending on whether the heaviest rainfall occurred in the first, second, third or fourth quarter of the total storm period. In each quartile group, 9 probability curves (from 10% to 90%, at 10% increments) were developed. From each of these probability mass curves, known as Huff curves, a design storm hyetograph can be constructed. Huff curves have been used previously as design hyetograph inputs to hydrologic models (Terstriep and Stall, 1974; U.S. Department of Agriculture, 1980; Bonta and Rao, 1986), and as a statistical input of storm temporal patterns in determining the flood probability distribution (Beran, 1973). However, Huff curves do not represent actual patterns with a certain probability of exceedance but rather they generate the statistics of an ensemble of patterns. In other words, they exhibit less variability than the patterns of individual storms.

Another probabilistic description of temporal patterns has been developed by Robinson and Sivapalan (1997) using normalised mass curves to describe temporal patterns of rainfall intensity. Normalised mass curves of observed storms are represented by beta distributions. The model employs a multiplicative structure to generate mass curve ordinates as a function of random numbers drawn from the beta distribution.

If Monte Carlo simulation is used in determining the derived flood frequency curve, temporal patterns can be represented as a sample of design storms obtained for a specified duration and depth. This can be achieved by using rainfall disaggregation models or stochastic rainfall models. The disaggregation model described by Robinson and Sivapalan (1997) appears to offer considerable promise in terms of practical applicability.

While temporal patterns of observed storms exhibit a high degree of variability, it is desirable to find a parsimonious way to describe their variability. It is also important to investigate the effects of geographic location, storm type, duration, depth and seasonal variations on rainfall temporal patterns. Results of this investigation will help to group observed storms in a realistic way before developing the distributions of temporal patterns (Garcia-Guzman and Aranda-Oliver, 1993).

3.3.3 PROBABILITY DISTRIBUTION OF LOSS

The term 'loss' for an event is defined as the amount of precipitation that does not appear as direct runoff, and it should include all processes and factors involved in reducing total catchment rainfall to the rainfall excess that produces runoff during a flood event. The concept of loss includes moisture intercepted by vegetation (interception loss), percolated into soil (infiltration), retained by surface storage (depression storage), evaporated (evaporation loss). As these loss components are dependent on topography, soils, vegetation and climate, the losses exhibit both temporal and spatial variability during an event.

There are two main issues related to the development of probability distributions of losses: (i) selection of a loss model or indicator suitable to Australian design practice; and (ii) selection of a method for deriving the probability distribution of losses.

Loss model

Many loss models do not account for the interception, depression storage and transmission losses directly, all the loss is simply treated as infiltration into the soil. The reduction of infiltration capacity with time is expressed by empirical equations (such as Horton) or by more physically based equations (such as Phillip and Green Ampt equations). Most of the previous derived distribution studies used this type of loss representation, e.g. Eagleson (1972), Russell et al. (1979) and Hebson and Wood (1982) assumed a constant infiltration capacity during a rainfall event; Diaz-Granados et al. (1984), Shen et al. (1990), Sivapalan et al. (1990) and Cadavid et al. (1991) used Phillip's equation to express the reduction of infiltration capacity with time. Some investigators (e.g. Beran, 1973; Fontaine and Potter, 1993) used a catchment wetness index.

In design practice, the use of simplified lumped conceptual loss models is preferred over the mathematical equations because of their simplicity and ability to approximate catchment runoff behaviour. This is particularly true for design loss which is probabilistic in nature and for which complicated theoretical models may not be required. Such conceptualised models do not consider the spatial variability or the real temporal pattern of storm losses; the model parameters are estimated using the total catchment response i.e. runoff.

In Australia, the most commonly adopted conceptual loss model is the initial loss-continuing loss model (I. E. Aust., 1958, 1977, 1987; Hill et al., 1996a, b). For a specific part of the catchment, the initial loss is the loss that occurs prior to the commencement of surface runoff, and can be considered to be composed of the interception loss, depression storage and infiltration that occurs before the soil surface saturates. The continuing loss is the average rate of loss throughout the remainder of the storm. For different parts of the catchment, the division between initial loss and continuing loss would occur at different times, depending on the spatial distribution of interception, depression storage and infiltration characteristics. For larger heterogeneous catchments, the link between initial loss and continuing loss values and catchment characteristics is therefore less direct. Another conceptual loss model which might be an alternative (particularly for urban catchments) to the initial loss-continuing loss model is the initial loss-proportional loss model. Another conceptual loss model used in some derived distribution studies is the United States SCS curve number method. In the limited application in Eastern Australia, this method has not performed well (Nandakumar et al., 1994).

From the consideration of simplicity, ability to approximate catchment runoff behaviour and wider application in Australian design practice, it appears that the *initial loss-continuing loss model* has the greatest potential for the present joint probability study. It may be considered sufficient to treat only one of the two variables in the initial loss-continuing loss model as a stochastic variable. It is thus proposed to derive a probability distribution for initial loss, while continuing loss will be represented by a single (deterministic) design value.

Probability distribution:

Examples of work done in determining probability distributions of losses in previous derived distribution studies are given below:

- Beran (1973) developed a probability distribution for the catchment wetness index, calculated from the soil moisture deficit and a 5-day antecedent precipitation index using observed daily rainfall data.
- Haan and Schulze (1987) derived a probability distribution of S (maximum potential abstraction from rainfall) from observed data using a simple relationship between S, runoff Q and curve number (Soil Conservation Service, 1972).

- Fontaine and Potter (1993) derived the probability distribution of antecedent soil moisture from the observed daily rainfall data.
- Overney et al. (1995) used the SCS abstraction method (Soil Conservation Service, 1972) to describe losses. The probability distribution of curve number values was derived from a large sample of observed rainfall-runoff events.

For most of the derived distribution studies, probability distributions of loss indicators have been developed using observed data. This appears to be the most promising method but care needs to be taken that the procedure used for the selection of observed events does not introduce bias into the resulting distribution. The data used by Hill et al. (1996a, b) in deriving design losses for south-east Australian catchments may be of particular use for this study.

The probability distribution of losses may be obtained using the continuous modelling approach. For example, Pearse (1997) suggested that the probability distribution of initial loss could be derived from a deterministic relationship between initial loss and catchment wetness where the catchment wetness could be obtained from continuous modelling.

3.4 CORRELATION BETWEEN FLOOD PRODUCING VARIABLES

The current design event approach recognises the correlation between various flood producing variables, e.g. the intensity-frequency-duration (IFD) curves in ARR (I.E. Aust., 1987) incorporate a dependence between rainfall intensity and duration; ARR temporal patterns capture dependence between temporal pattern and rainfall duration and intensity (to some degree).

Most of the previous studies on the joint probability approach assumed the flood producing components to be independent. The degree of dependence between various flood producing components (i.e. rainfall intensity, rainfall duration, rainfall temporal pattern and losses) is discussed below and summarised in Table 3.2.

Most of these studies either assumed rainfall intensity and duration as independent random variables or used a fixed value of rainfall duration; however, it is well known that rainfall intensity decreases as duration increases. Sivapalan et al. (1996) proposed a method of specifying the joint distribution of rainfall intensity and duration which considers IFD curves as conditional distributions and distribution of storm duration as marginal distribution. The advantage of using IFD curves is that they explicitly incorporate the dependence between rainfall intensity and duration. Bloschl and Sivapalan (1997) mentioned that "the case of independent intensity-duration gives vastly steeper flood frequency curves than the case of dependent intensity-duration". In the independent case, combinations of large durations and large intensities are possible which produce excessively large peaks at small AEPs. This is not very realistic from a physical point of view as long duration events rarely have the same average intensities as short events. Bloschl and Sivapalan (1997) argued that independence of intensity and duration steepens the flood frequency curve which compensates for the inadequate representation of non-linear runoff generation, multiple storms and within-storm temporal pattern.

The correlation between rainfall intensity and temporal pattern is such that for higher intensity the temporal pattern tends to be more uniform (I.E. Aust., 1987). It has been allowed for in ARR by specifying temporal patterns for 20 different durations in the range from 6 minutes to 72 hours. But the degree of correlation between rainfall intensity and temporal pattern seems to be not very high (graded as 'low-medium' in Table 3.2).

Hill et al. (1996a, b) found no clear dependence between loss and rainfall severity. However, they noted that design initial loss increases with the duration of the burst. This is because, for the shorter duration bursts, a large proportion of bursts are embedded within longer duration storms and a portion of the initial loss is satisfied by pre-burst rainfall; as the duration increases, more bursts represent complete storms. The dependence between design loss and temporal pattern requires further investigation.

When correlations between various flood producing components are considered, the statistical treatment of the process becomes highly complicated, particularly when the analytical approach of derived distributions is used. In the approximate methods, dependence among variables can be considered relatively easily. It appears that at least the correlations between variables graded as 'high' and 'medium-high' in Table 3.2 should be considered in a joint probability approach aiming at an improved design flood estimation technique.

Table 3.2 Degree of dependence between various flood producing variables

	Rainfall duration	Temporal pattern	Loss rate
Rainfall intensity	high	low-medium	low-medium
Rainfall duration		medium-high	medium-high
Temporal pattern			?

4. A PRACTICAL FLOOD ESTIMATION TECHNIQUE BASED ON THE JOINT PROBABILITY APPROACH

4.1 AUSTRALIAN DESIGN PRACTICE

The rainfall-based design flood estimation techniques used in Australia are all based on the design event approach. The limitations of this approach have been mentioned in Section 2.2.1.

From a practical point of view, it would be desirable for any new design flood estimation approach to exploit existing flood estimation models and design data as far as possible, thus making maximum use of existing expertise and experience.

The rainfall runoff model commonly used in Australia is semi-distributed and non-linear, e.g. RORB (Laurenson and Mein, 1997). The particular advantages of this type of model are that the areal variation of rainfall inputs and losses, and the effects of varying flow distance to the catchment outlet are accounted for to a good extent. Of the model inputs, only rainfall depth is described by a probability distribution. Probabilistic inputs of other flood producing components and consideration of their interaction, as proposed by this project, could reduce bias, uncertainty and inconsistencies associated with the design flood estimates.

The IFD curves of the rainfall intensity and the temporal patterns in ARR (I. E. Aust., 1987) are based on intense bursts of rainfall rather than complete storms, whereas design losses in ARR have been derived using complete storms. Hill et al. (1996a, b) derived design losses for Victorian catchments based on rainfall bursts. To use available design data (e.g. ARR design rainfall data, Hill et al., 1996a, b loss information) with the proposed joint probability study, consistency needs to be maintained in the selection of the storm. Alternatively, the effects of different storm definitions (complete storms or bursts) on the results need to be investigated.

4.2 COMPONENTS OF A MODELLING FRAMEWORK BASED ON THE JOINT PROBABILITY APPROACH

The development of a design flood estimation technique based on the joint probability approach involves three major steps: (i) selection of flood producing variables to be considered as random variables and determination of their probability distributions; (ii) selection of suitable runoff production and runoff routing models ; and (iii) selection of a mathematical framework within which the first two components are combined to determine the derived flood frequency distribution. These steps are discussed below with a view to work out a methodology that is likely to lead to an improved design flood estimation technique that can be easily applied in Australian design practice.

4.2.1 FLOOD PRODUCING VARIABLES AND THEIR DISTRIBUTIONS

The flood producing variables that appear to be important and need to be considered as random variables have been identified in Section 3.2. These are rainfall event duration, average rainfall intensity for the event over a catchment, rainfall temporal pattern, and losses. The derivation of probability distributions for the rainfall variables is discussed in this section, while the loss variables are discussed in Section 4.2.2.

In order to treat rainfall duration as a random variable, a rainfall event needs to be defined in such a way that both rainfall duration and average rainfall intensity become random variables. A 'complete storm' generally satisfies this requirement and can be defined as a period of non-zero rainfall that is seperated from another storm by a specified period (e.g. 2 hours) of zero rainfall. The duration of a complete storm is a random variable, thus IFD curves based on complete storms are required for use with the joint probability approach. However, the IFD curves used in current design practice (e.g. ARR IFD curves) are not based on complete storms, but on periods of intense rainfall within complete storms (called bursts). A key issue is whether the existing ARR IFD curves can be used with the proposed joint probability approach.

An alternative to the analysis of complete storms would be the use of a rainfall event that considers only the highest intensity part of a storm, i.e. a storm burst. However, the durations of the bursts in the ARR analysis were predetermined rather than random. Hence, it would be necessary to consider a new burst definition that will produce randomly distributed burst durations. These new bursts may be referred to as 'storm-cores'. For each complete storm, there would be one storm-core; it is the burst of that duration which is associated with the greatest relative average intensity compared to a threshold intensity. The threshold intensity may be fixed or a function of burst duration

The relationship between the ARR IFD curves and complete storm or storm-core IFD curves needs to be investigated to examine whether they are similar. In case of significant dissimilarity, if some meaningful relationship could be established between these IFD curves, then the ARR IFD curves could be used to estimate the IFD curves required for the joint probability approach.

Most of the previous studies (e.g. Eagleson, 1972; Becciu et al., 1993; Sivapalan et al., 1996) have used empirical areal reduction factors (ARFs) to convert point rainfalls into catchment rainfalls. Recently, Siriwardena and Weinmann (1996) have derived ARFs for Victoria; these were significantly lower than the ARR (I. E. Aust, 1987) values and have been recommended for adoption for Victorian catchments. Hence, these ARFs can be used in the proposed study for a deterministic transformation of point rainfalls into catchment rainfalls.

The probability distribution of temporal patterns can be obtained from the observed rainfall data using the same rainfall event definition as used in obtaining probability distributions of rainfall duration and intensity. Stochastic rainfall disaggregation models of the type described by Robinson and Sivapalan (1997) appear to offer promise for practical application. The influence of geographic location, season, rainfall duration and intensity on the variation of patterns needs to be further investigated.

4.2.2 RUNOFF PRODUCTION AND RUNOFF ROUTING MODELS

Most of the previous derived distribution studies used a loss model based on some mathematical equations e.g. Horton's equation, Phillip's equation (Section 3.3.2). However, in design practice, the use of simplified lumped conceptual loss models is preferred over the mathematical equations, particularly for design losses which is probabilistic in nature and for which complicated theoretical models may not be required. For the proposed study, the *initial loss-continuing loss model* has been identified as an appropriate model in Section 3.3.3. *Initial loss* is to be treated as a stochastic variable, and its probability distribution can be obtained by analysing observed rainfall and runoff data, in a similar fashion as described by Hill et al (1996a,b).

The use of a loss model and the within-rainfall-event temporal pattern allows the estimation of a rainfall excess hyetograph from the total rainfall depth. A catchment response model (runoff routing model) is then needed to convert the rainfall excess hyetograph into a surface runoff hydrograph. The models previously used in derived flood frequency distribution studies have been discussed in Section 3.1. Frequently used models are (a) Eagleson's (1972) kinematic wave model; (b) models based on the geomorphologic unit hydrograph (GUH); and (c) unit hydrograph models.

A semi-distributed and non-linear type of catchment response model (such as RORB, URBS) is commonly used in Australia. This type of model appears to be preferable to the models mentioned above because this model, being distributed in nature, can account for the areal variation of rainfall and losses, and can consider catchment non-linearity. Hence, RORB (Laurenson and Mein, 1997) or URBS (Carroll, 1994) offers greater potential than the other models mentioned above. There is also a considerable body of experience available in Australia on appropriate parameter values for different types of catchments.

4.2.3 MATHEMATICAL FRAMEWORK

The mathematical frameworks adopted in the previous derived distribution studies have been classified into two groups (Section 3.1): (a) analytical methods and (b) approximate methods.

The analytical approach has been adopted by many investigators (e.g. Eagleson, 1972; Wood, 1976; Hebson and Wood, 1982; Diaz-Granados et al., 1984; Beven, 1986, Cadavid et al., 1991, Sivapalan et al., 1990, 1996). This method normally involves evaluation of complicated integrals which makes it difficult to apply in practical situations. Also it requires very simplified assumptions which may not be valid in real catchments. Thus, the analytical approach is unlikely to lead to a technique that is flexible enough to be used in design practice.

The approximate methods have been adopted by some investigators as mentioned in Section 3.1.2. Many of these use discrete probability distributions to describe flood producing variables, and utilise the Theorem of Total Probability to calculate flood probability (e.g. Laurenson, 1974; Russell et al., 1979; Ahern and Weinmann, 1982; Laurenson and Pearse, 1991; Fontaine and Potter, 1993). This method has the potential to be applied in design practice. The simulation techniques have been adopted by some investigators e.g. Beran (1973), Muzik (1993), Bloschl and Sivapalan (1997). In these techniques, a large number of generated flood peaks are used to construct derived flood frequency distributions. From a practical application point of view, this technique has also greater potential than the analytical approach.

It appears that the approximate methods (discretization of the continuous variables and application of the Total Probability Theorem or sampling from continuous distributions of the variables), coupled with a method of considering dependence between the flood producing variables, would be the most appropriate mathematical framework to lead to improved techniques that could be easily applied in practice. The essential components of such a technique are presented in Figure 4.1.

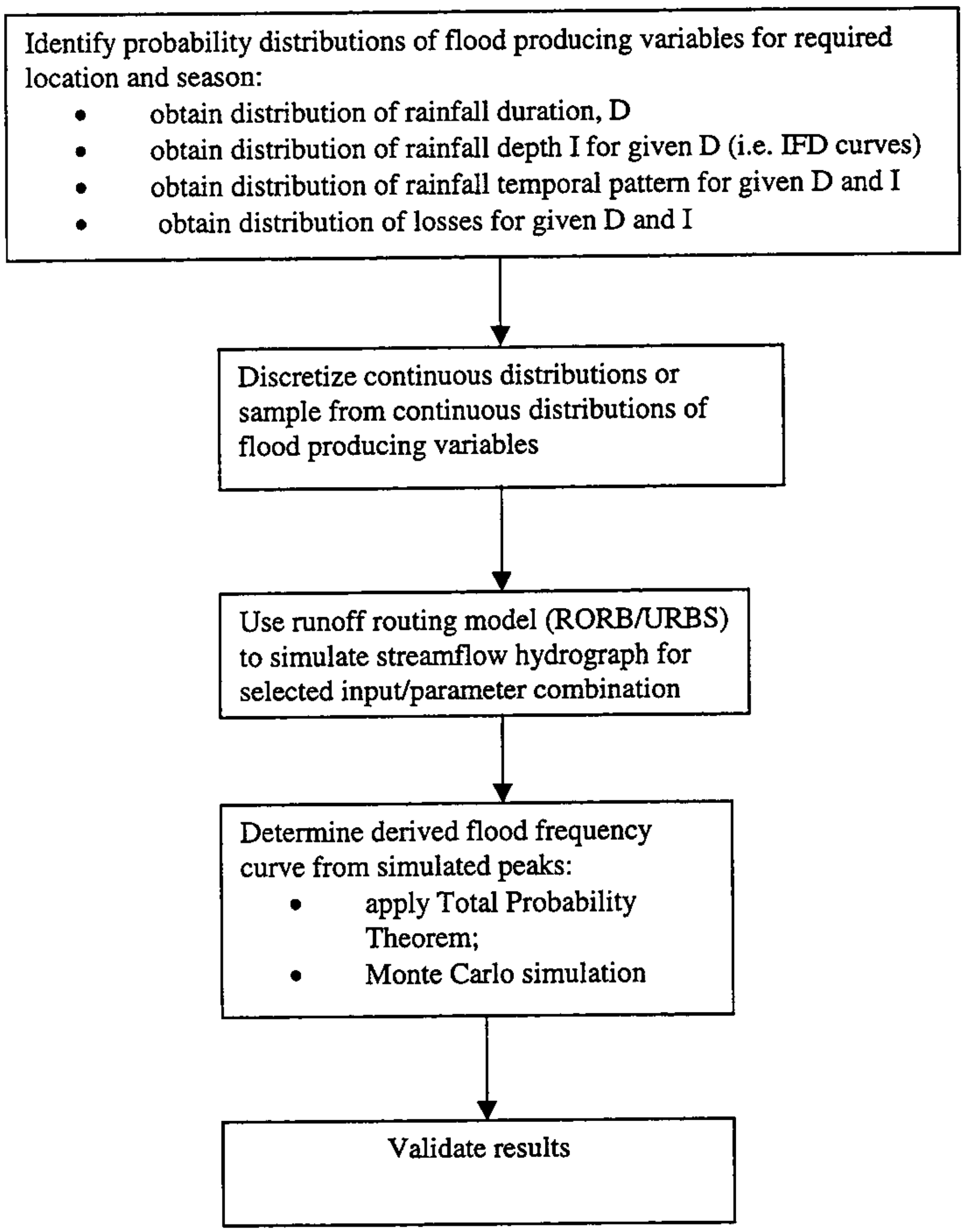

Figure 4.1 Components of the recommended modelling framework

4.3 RESEARCH ISSUES

This review has identified a number of research issues that need to be addressed to develop a *practical* design flood estimation technique based on the joint probability approach. It is proposed that these issues be addressed through the following research tasks:

(i) identification of appropriate rainfall event definitions for complete storms and storm-cores and examination of their validity for the intended purposes;

(ii) derivation of probability distributions for the following flood producing variables for both complete storms and storm-cores:
  - rainfall duration;
  - average rainfall intensity;
  - rainfall temporal pattern; and
  - initial loss;

(iii) examination of the dependence of flood producing variables with geographic location and season, as well as interdependence between variables;

(iv) determination of the relationship of distribution parameters described in (ii) with existing design data;

(v) applicability of existing flood estimation models with new method and modification as required;

(vi) development of a new computational framework for the joint probability approach with sufficient flexibility to allow easy application in practical situations; and

(vii) demonstration of the relative performance of new method through application to test catchments and comparison with existing methods.

5. CONCLUSION

This review has examined the current state of research and practice in joint probability approaches to design flood estimation with a particular emphasis on the *applicability* of the methods in design practice. The main findings of this review are that:

(i) The current design event approach is likely to introduce significant bias and uncertainty in design flood estimates. The joint probability approach has the potential to reduce this bias and uncertainty significantly, and will lead to more consistent design flood estimates.

(ii) Most of the previous studies with joint probability approaches have been found to be limited to experimental and small catchments; the complexity of the model, difficulties in parameter estimation and limited flexibility of the approach have made it difficult to apply the method in design practice.

(iii) The components of a *practical* flood estimation modelling framework based on the joint probability approach have been identified:

- the important flood producing variables are rainfall duration, rainfall intensity, rainfall temporal pattern and initial loss – these need to be described by probability distributions rather than representative design values;
- an initial loss-continuing model combined with a semi-distributed non-linear runoff routing model like RORB and URBS would be appropriate to generate flood hydrographs for different input/parameter combinations; and
- Monte Carlo simulation and application of the Total Probability Theorem appear to be suitable methods for determining the derived distributions from a practical application view point.

Based on these findings, a list of proposed research tasks has been prepared to be addressed in CRCCH Sub Project FL1.1 (Section 4.3).

REFERENCES

Ahern, P. A. and Weinmann, P. E. (1982). Considerations for Design Flood Estimation Using Catchment Modelling. Hydrology and Water Resources Symposium, Melbourne, Australia, pp.44-48.

Askew, A. J. (1975). Variations in Estimates of Design Floods. Hydrology Symposium, Institution of Engineers Australia Ed., National Conference Publication, pp. 26-30.

Bates, B. C. (1994). Regionalisation of Hydrological Data: A Review. Cooperative Research Centre for Catchment Hydrology (CRCCH) Report 94/5. Dept. of Civil Eng., Monash University, Australia.

Becciu, G., Brath, A. and Rosso, R. (1993). A Physically Based Methodology for Regional Flood Frequency Analysis. Proc. ASCE Hydraul. Div. San Francisco, July 1993. pp. 461- 466.

Benjamin, J. R. and Cornell, C. A. (1970). Probability, Statistics, and Decision for Civil Engineers. McGraw-Hill Book Company.

Beran, M. A. (1973). Estimation of Design Floods and the Problem of Equating the Probability of Rainfall and Runoff. Symposium on the Design of Water Resources Projects with Inadequate Data, Madrid, Spain, pp. 33-50.

Beran, M. A. and Sutcliffe, J. V. (1972). An Index of Flood-Producing Rainfall Based on Rainfall and Soil Moisture Deficit. Journal of Hydrology, 17, 229-236.

Beven, K. J. (1986). Hillslope Runoff Processes and Flood Frequency Characteristics. Hillslope Processes. Edited by A. D. Abrahams, Allen and Unwin, 181-202.

Bloschl, G. and Sivapalan, M. (1997). Process Controls on Flood Frequency. 2. Runoff Generation, Storm Properties and Return Period. Centre for Water Research Environmental Dynamics Report, ED 1159 MS, Department of Civil Eng., The University of Western Australia.

Bonta, J. V. and Rao, A. R. (1986). Estimation of Peak Runoff Rates from Small Agricultural Watershed. (Technical Report No. CE-HSE-86-3). School of Civil Engineering, Purdue University, West Lafayette, Indiana.

Brown, J. A. H. (1982). A Review of Flood Estimation Procedures. Proceeding of the Workshop on Spillway Design, Department of Natural Development and Energy Ed., AWRC Conference Series, pp. 84-108.

Cadavid, L. J., Obeysekera, T. B. and Shen, H. W. (1991). Flood Frequency Derivation from Kinematic Wave, J. Hydraul. Eng. ASCE, 117(4), 489-510, 1991.

Carroll, D. G. (1994): The BCC Catchment Management Runoff Routing Model Manual, Version 3.3, Brisbane city Council, Brisbale, Australia.

Clark, C. O. (1945). Storage and the Unit Hydrograph. Transaction of ASCE., 110, 1419-1488.

Consuegra, D., Meylan, P. and Musy, A. (1993). A Probabilistic Approach to Estimate Design Parameters for Flood Control Projects. San Francisco, A. H. Division Ed., pp. 455-460.

Cordery, I., Pilgrim, D. H. and Rowbottom, I. A. (1984). Time Patterns of Rainfall for Estimating Design Floods on a Frequency Basis. Water Science Tech., 16, 155-165.

Crawford, N. H. and Linsley, R. K. (1966). Digital Simulation in Hydrology: Stanford Watershed Model IV. (Technical Report 39). Department of Civil Engineering, Stanford University.

CRCCH (1995). Catchword. (Newsletter of the CRCCH No. 35, December 1995). Cooperative Research Centre for Catchment Hydrology.

Diaz-Granados, M. A., Valdes, J. B. and Bras, R. L. (1984). A Physically Based Flood Frequency Distribution. Water Resources Research, 20, 7, 995-1002.

Dunsmore, S. J., Schulze, R. E. and Schmidt, E. J. (1986). Antecedent Soil Moisture in Design Stormflow Estimation. (ACRU Report No. 23). Department of Agricultural Engineering.

Durrans, S. R. (1995). Total Probability Methods for Problems in Flood Frequency Estimation, in Statistical and Bayesian Methods in Hydrological Sciences, Proceedings of the International Conference in Honour of Jacques Bernier, Sep. 11-13, Chapter III, UNESCO, Paris.

Eagleson, P. S. (1972). Dynamics of Flood Frequency. Water Resources Research, 8, 4, 878-898.

Eagleson, P. S. (1978). Climate, Soil and Vegetation 5. A Derived Distribution of Storm Surface Runoff. Water Resources Research, 14, 5, 741-748.

Edwards, D. R. and Haan, C. T. (1989). Incorporating Uncertainty into Peak Flow Estimates. Transactions of the ASAE, 32, 113-119.

Feldman, A. D. (1979). Flood Hydrograph and Peak Flow Frequency Analysis. (Technical Paper No. 62). The Hydrologic Engineering Centre.

Fontaine, T. A.and Potter, K. W. (1993). Estimating Exceedance Probabilities of Extreme Floods. San Francisco, A. H. Division Ed., pp. 635-640.

Freund, J. E. (1961). A Bivariate Extension of the Exponential Distribution. Journal of the American Statistical Association, 56, 971-977.

Goyen, A. G. (1983). A Model to Statistically Derive Design Rainfall Losses. Hydrology and Water Resources Symposium, Hobart, Australia, pp. 220-225.

Gupta, V.K., Waymire, E. and Wang, C. T. (1980). A Representation of an Instantaneous Unit Hydrograph from Geomorphology. Water Resour. Res., 16(5): 855-862.

Gutknecht, D. (1977). Ein Computermodell zur Erzeugung einer Folge von Regenereignissen, Tech. Report, Inst. f. Hydraulik, Technical University of Vienna.

Guzman-Garcia, A. and Aranda-Oliver, E. (1993). A Stochastic Model of Dimensionless Hyetograph. 29(7): 2363-2370.

Haan, C. T. (1977). Statistical Methods in Hydrology. First Ed., The Iowa State University Press.

Haan, C. T. and Edwards, D. R. (1988). Joint Probability Estimates of Return Period Flows. Transactions of the ASAE, 31(4):July-August.

Haan, C. T. and Schulze, R. E. (1987). Return Period Flow Prediction with Uncertain Parameters. Transactions of the ASAE, 30(3):May-June, 665-669.

Hammersley, I. M. and Handscomb, D. C. (1964). Monte Carlo Methods. Wiley, New York.

Hashino, M. (1986). Stochastic Formulation of Storm Pattern and Rainfall Intensity-Duration Curve for Design Flood. International Symposium on Flood Frequency and Risk Analysis, Barton Rouge, U.S.A., V. P. Singh Ed., D. Reidel Publishing Company, pp. 303-315.

Hebson, C. and Wood, E. F. (1982). A Derived Flood Frequency Distribution Using Horton Order Ratios. Water Resources Research, 18, 5, 1509-1518.

Haan, C. T. and Wilson B. N. (1987). Another Look at the Joint Probability of Rainfall and Runoff. Hydrologic Frequency Modeling, edited by V. P. Singh, Reidel Publishing Company, pp. 555-569.

Hill, P. I. (1994). Catalouge of Hydrologic Data for Selected Victorian Catchments. CRC for Catchment Hydrology Working Document No 94/1. Department of Civil Engineering, Monash University, Australia.

Hill, P. I. and Mein, R. G. (1996). Incompatibilities between Storm Temporal Patterns and Losses for Design Flood Estimation. Hydrology and Water Resources Symposium 1996, Hobart, Tasmania, 2, pp. 445-451.

Hill, P. I., Mein, R. G., Weinmann, P. E. (1996a). Testing of Improved Inputs for Design Flood Estimation in South-Eastern Australia. CRC for Catchment Hydrology Report 96/6, Department of Civil Engineering, Monash University, Australia.

Hill, P. I., Maheepala, U. K., Mein, R. G. and Weinmann, P. E. (1996b). Empirical Analysis of Data to Derive Losses for Design Flood Estimation in South-Eastern Australia. CRC Report No 96/5, CRC for Catchment Hydrology, Department of Civil Engineering, Monash University, Australia.

Hjelmfelt, A. T., Jr. (1980). Empirical Investigation of Curve Number Technique. Proc. Am. Soc. of Civil Engineers, Vol. 106(HY9). pp. 1471-1476.

Hoang, T. (1997). A Joint Probability Approach to Rainfall-based Design Flood Estimation. Project Proposal for PhD research, Dep. of Civil Eng., Monash University, Clayton, Victoria, Australia.

Huber, W. C., Cunningham, B. A.and Cavender, K. A. (1986). Continuous SWMM Modelling for Selection of Design Events. Comparison of Urban Drainage Models with Real Catchment Data, Dubrovnik, Yugoslavia, C. Maksimocic & M. Radojkovic Ed.

Huff, F. A. (1967). Time Distribution of Rainfall in Heavy Storms. Water Resources Research, 3, 4, 1007-1019.

Institution of Engineers Australia (1958). Australian Rainfall and Runoff. Institution of Engineers, Australia.

Institution of Engineers Australia (1977). Australian Rainfall and Runoff. Institution of Engineers, Australia.

Institution of Engineers Australia (1987). Australian Rainfall and Runoff. Institution of Engineers, Australia.

James, W.and Robinson, M. (1986). Continuous Deterministic Urban Runoff Modelling. Comparison of Urban Drainage Models with Real Catchment Data, Dubrovnik, Yugoslavia, C. Maksimovic & M. Radojkovic Ed.

James, W.and Robinson, M. A. (1982). Continuous Models Essential for Detention Design. Engineering Foundation Conference on Storm Water Detention Facilities Planning Design Operation and Maintenance, American Society of Civil Engineers, pp. 163-175.

Keifer, C. J.and Chu, H. H. (1957). Synthetic Storm Pattern for Drainage Design. Journal of the Hydraulics Division, 83, HY4, 1-25.

Kennedy, M. R., Turner, L., H.and Canterford, R. P. (1991). Temporal Distributions Within Rainfall Bursts. (Hydrology Report Series No. 1). Hydrometeorological Advisory Service.

King County (Washington) (1995). Surface Water Design Manual. Washington, King County.

Kuczera, G. (1994). NLFIT - A Bayesian Nonlinear Regression Program Suite. New South Wales: The University of Newcastle.

Larson, C. L.and Reich, B. M. (1972). Relationship of Observed Rainfall and Runoff Recurrence Intervals. Water Resources Publications, pp. 34-43.

Laurenson, E. M. (1973). Effects of Dams on Flood Frequency. International Symposium On River Mechanics, Bangkok, Thailand, I. A. F. H. Research Ed., pp. 133-146.

Laurenson, E. M. (1974). Modelling of Stochastic-Deterministic Hydrologic Systems. Water Resources Research, 10, 5, 955-961.

Laurenson, E. M. and Mein, R. G. (1997). RORB Version 4 Runoff Routing Program User Manual, Dep. of Civil Eng., Monash University. pp. 186.

Laurenson, E. M.and Pearse, M. A. (1991). Frequency of Extreme Rainfall and Floods. International Hydrology & Water Resources Symposium, Perth, pp. 392-399.

Leclerc, G. and Schaake, J. (1972). Derivation of Hydrologic Frequency Curves, Tech. Rep. 142. Ralph Parsons Lab. for Water Resour. and Hydrodyn., MIT, Cambridge, Mass., 1972.

Linsley, R.and Crawford, N. (1974). Continuous Simulation Models in Hydrology. Geophysical Research Letters, 1, 1, 59-62.

Linsley, R. K., Kohler, M. A.and Paulhus, J. L. H. (1988). Hydrology for Engineers. McGraw-Hill Book Company.

Lumb, A. M.and James, L. D. (1976). Runoff files for flood hydrograph simulation. Journal of the Hydraulics Division, ASCE, 1515-1531.

Milston, A. K. (1979). The Influence of Temporal Patterns of Design Rainfall on Peak Flood Discharge. Masters of Engineering Science, University of New South Wales.

Moughamian, M. S., McLaughlin, D. B. and Bras, R. L. (1987). Estimation of Flood Frequency: An Evaluation of two derived distribution procedures. Water Resour. Res., 23(7), 1309-1319.

Muzik, I. (1993). Derived, physically based distribution of flood probabilities. Extreme Hydrological Events: Precipitation, Floods and Droughts (Proceedings of the Yokohama Symp., July 1993, IAHS Publ. No. 213. pp. 183-188.

Muzik, I. (1994). Understanding flood probabilities. Stochastic and Statistical Methods in Hydrology and Environmental Engineering, Vol. 1, 199-207, K. W. Hipel (ed), Kluwer Academic Publishers, Netherlands.

Muzik, I. and Beersing, A. K. (1989). Stochastic-Deterministic Nature of an Elemental Rainfall Runoff Process. Water Resour. Res. Vol. 25, No. 8. pp. 1805-1814.

Nandakumar, N., Mein, R.G. and Siriwardena, L. (1994). Loss Modelling For Flood Estimation - A Review. CRCCH Report No 94/4, Dept. of Civil Eng., Monash University.

Nathan, R. J. (1992). The Derivation of Design Temporal Patterns for use with Generalised Estimates of Probable Maximum Precipitation. Civil Eng. Trans., I. E. Aust., CE34 (2): 139-150, 1992.

Nathan, R. J. and Weinmann, P. E. (1993). Low Flow Atlas for Victorian Streams. Division of Water Resources. Department of Conservation and Natural Resources, 170pp.

Natural Environment Research Council (1975). Flood Studies Report, NERC, London.

Naylor, T. H. et al. (1966). Computer Simulation Techniques. John Wiley & Sons, Inc.

Ott, R. F. and Linsley, R. K. (1972). Streamflow frequency using stochastically generated hourly rainfall. Proc. Int. Symp. On Uncertainties in Hydrologic and Water Resources Systems. Vol. 1, 2.8: 1-15, December 1972.

Overney, O. et al. (1995). Estimating Expected Flood Values with Monte-Carlo Methods. Bernier Conference, 1995, Paris.

Institution of Engineers Australia (1958). Australian Rainfall and Runoff. Institution of Engineers, Australia.

Pearse, M. A. (1997). A Joint Probability Approach to Estimating the Frequency of Extreme Precipitation for Flood Estimation. Unpublished PhD thesis, Monash University.

Pearse, M. A. and Laurenson, E. M. (1997). Real Probabilities for Real Floods. ANCOLD Conference on Dams, 1997, Australian National Committee on Large Dams.

Philip, J. R. (1957). The theory of infiltration, 1, The infiltration equation and its solution, Soil Sci., 83, 345-357.

Philip, J. R. (1969). Theory of Infiltration. Advances in Hydroscience, 5, 215-296.

Pilgrim, D. H.and Cordery, I. (1975). Rainfall Temporal Patterns for Design Floods. Journal of the Hydraulics Division, 101, HY1, 81-95.

Raines, T. H. and Valdes, J. B. (1993). Estimation of flood frequencies for ungauged catchments. J. Hydraul. Eng., Am. Soc. Civ. Eng., 119(10), 1138-1154.

Reich, B. M. (1970). Flood Series Compared to Rainfall extremes. Water Resour. Res., 1655-1667.

Robinson, J. S. and Sivapalan, M. (1997). Temporal Scales and Hydrological Regimes: Implications for flood frequency scaling. Water Resour. Res. 33(12)2981-2999.

Rodriguez-Iturbe, I. and Valdes, J.B. (1979). The geomorphologic structure of hydrologic response. Water Resour. Res., 15(6), 1409-1420.

Rodriguez-Iturbe, I., Gonzales-Sanabria, M. and Bras, R. L. (1982a). A Geomorphoclimatic Theory of the Instantaneous Unit Hydrograph. Water Resour. Res., 18(4):877-886.

Rodriguez-Iturbe, I., Gonzales-Sanabria, M. and Caamano, G. (1982b). A Geomorphoclimatic Theory of the Instantaneous Unit Hydrograph. Water Resour. Res., 18(4):887-903.

Rubinstein, R. V. (1981). Simulation and the Monte Carlo Method. John Wiley & Sons.

Russell, S. O. (1977). A New Method for Computing Design Flows for Urban Drainage. Proceedings of Hydrology Symposium, Edmonton, Canada, 11, pp. 188-197.

Russell, S. O., Kenning, B. F. I.and Sunnell, G. J. (1979). Estimating Design Flows for Urban Drainage. Journal of the Hydraulics Division, 105, 43-52.

Schaake, J. C., Geyer, J. C. and Knapp, J. W. (1967). Experimental Examination of the Rational Method. Proc. Am. Soc. of Civil Engineers, Vol. 93(HY6), pp. 353-370.

Shen, H. W., Koch, G. J. and Obeysekera, J. T. B. (1990). Physically Based Flood Features and Frequencies. J. Hydraul. Eng., Vol. 116, No. 4. pp. 494-514.

Siriwardena, L. and Weinmann, P. E. (1996). Derivation of Areal Reduction Factors for Design Rainfalls in Victoria. CRC for Catchment Hydrology Report 96/4, Dept. of Civil Eng., Monash University, pp. 60.

Sivapalan, M., Bloschl, G. and Gutknecht, D. (1996). Process Controls on Flood Frequency. 1. Derived Flood Frequency. Centre for Water Research Environmental Dynamics Report, ED 1158 MS, Department of Civil Eng., The University of Western Australia.

Sivapalan, M., Beven, K.J. and Wood, E.F. (1990). On Hydrologic Similarity, 3, A Dimensionless Flood Frequency Model Using a Generalised Geomorphic Unit Hydrograph and Partial Area Runoff Generation, Water Resour. Res., 26(1), 43-58.

Sode, R. L. (1972). Annual Report 1972. Ingham County, Michigan.

Soil Conservation Service (1972). Section 4: Hydrology. National Engineering Handbook Washington D.C.: U.S. Department of Agriculture.

Tavakkoli, D. (1985). Simulation von Hochwasserwellen aus Niederschlagen. PhD Thesis, Inst. f. Hydraulik, Technical University of Vienna.

Terstriep, M. L.and Stall, J. B. (1974). The Illinois Urban Drainage Area Simulator, ILLUDAS. (Bulletin 58), Illinois State Water Survey.

Troch, P. A., Smith, J. A., Wood, E. F. and Troch, F. De. (1994). Hydrologic controls of large floods in a small basin: Central Appalachian case study, J. Hydrol., 156, 285-309.

U.S. Department of Agriculture (1980). CREAMS - A Field-Scale Model for Chemicals, Runoff, and Erosion from Agricultural Management Systems. (Conservation Research Report No. 26). Science and Education Adm.

Viessman, W. J., Lewis, G. L.and Knapp, J. W. (1989). Introduction to Hydrology. Harper Collins Publishers.

Walpole, R. E.and Myers, R. H. (1993). Probability and Statistics for Engineers and Scientists. Fifth Ed., Prentice-Hall International, Inc.

Walsh, M. A., Pilgrim, D. H. and Cordery, I. (1991). Initial Losses for Design Flood Estimation in New South Wales. International Hydrology and Water Resources Symposium, Perth, I. E. Aust. Nat. Conf. Pub. No. 91/19, pp. 283-288.

Weinmann, P. E. (1994). On the Origin and Variation of Skewness in Flood Frequency Distributions. Water Down Under '94, Adelaide, Australia, pp. 265-270.

Willing and Partners Pty Ltd (1994). RAFTS-XP Runoff Analysis & Flow Training Simulation with XP Graphical Interface. WP Software 1994.

Wood, E. F. (1976). An analysis of the effects of parameter uncertainity in deterministic hydrologic models. Water Resour. Res., 12(5), 925-932.

Wood, J. F.and Alvarez, K. (1982). Review of Pindari Dam Design Inflow Flood Estimate. Proceeding of the Workshop on Spillway Design, Department of Natural Development and Energy Ed., AWRC Conference Series, pp. 119-140.

Wood, E. F. and Hebson, C. S. (1986). On hydrologic similarity, 1, Derivation of Flood Frequency curve. Water Resour. Res., 22, 1549-1554.

Yen, B. C.and Chow, V. T. (1980). Design Hyetographs for Small Drainage Structures. Journal of the Hydraulics Division, 106, HY6, 1055-1077.

APPENDIX A

JOINT PROBABILITY APPROACH: STATISTICAL BASIS

A1 BASIC PROBABILITY CONCEPTS

The discussion given below mainly follows from Benjamin and Cornell (1970) and Walpole and Myers (1993)

Probability of union: The probability of an event which is the union of two events A and B, i.e. the probability of the occurrence of either A or B or both, denoted by $P(A \cup B)$, is the sum of their individual probabilities minus the probability of their joint occurrence (Benjamin and Cornell, 1970). That is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{A1}$$

If events A and B are mutually exclusive then $P(A \cap B)$ becomes zero thus Equation A1 reduces to:

$$P(A \cup B) = P(A) + P(B) \tag{A2}$$

Conditional probability and joint probability: Conditional probability is a concept of great practical importance, because it provides the capability of re-evaluating the probability of an event in light of additional information (Walpole and Myers, 1993). The conditional probability of the event A given that the event B has occurred, denoted by $P(A|B)$, is defined as the ratio of the probability of the intersection of A and B to the probability of the event B. That is:

$$P(A|B) = P(A \cap B)/ P(B) \tag{A3}$$

Here $P(A \cap B)$ is the probability of joint occurrence of the events A and B and called "joint probability of events A and B". In application, $P(B)$ and $P(A|B)$ often come from a study of the problem, whereas actually the joint probability $P(A \cap B)$ is desired; this is obtained as follows:

$$P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A) \tag{A4}$$

Independence of two events: If two physical events are not related in any way, the measure of the probability of one will not be changed if it is known that the other has occurred. This intuitive notion leads to the definition of probabilistic independence. Two events A and B are said to be independent if and only if

$$P(A|B) = P(A) \tag{A5}$$

From Equation A4 this definition of independence implies that

$$P(A \cap B) = P(A|B)\,P(B) = P(A)\,P(B) \tag{A6}$$

To generalise, if events A, B, C, . . . are mutually independent then their joint probability of occurrence is

$$P(A \cap B \cap C \ldots) = P(A)\,P(B)\,P(C) \ldots \tag{A7}$$

This is known *as multiplicative rule*, and plays an important role in the statistical hydrology. Expressed in words, if events are independent, the probability of their joint occurrence is simply the product of their individual probabilities of occurrence.

In flood hydrology, many of the flood causing components are assumed to be independent, but in reality, most of these are not completely independent. Without independence, the mathematical treatment of many hydrological phenomenon becomes highly complicated.

Total Probability Theorem: If B_i (i varies from 1 to n, where n is a positive integer) represents a set of events which satisfies the following two conditions: (i) mutually exclusive, i.e. $P(B_1 \cup B_2 \cup \ldots \cup B_n) = P(B_1) + P(B_2) + \ldots + P(B_n)$ and (ii) collectively exhaustive, i.e. $P(B_1 \cup B_2 \cup \ldots \cup B_n) = 1$, then the probability of another event A can be determined by using the Total Probability Theorem. This can be given as:

$$P(A) = P(A \cap B1) + P(A \cap B_2) + P(A \cap B_3) + \ldots P(A \cap B_n) \quad \text{(A8)}$$

This theorem represents the expansion of the probability of an event in terms of its conditional probabilities, conditioned on a set of mutually exclusive, collectively exhaustive events. It is often a useful expansion in problems where it is desired to compute the probability of an event A, since the terms in the sum may be more readily obtainable than the probability of A itself (Benjamin and Cornell, 1970). It is considered as one of the workhorses in probability applications (Kuczera, 1994).

The Theorem of Total Probability, expressed in one dimension by Equation A8, can also be expanded to two or more dimensions. For example, in three dimensions B, C, D, the theorem is written as follows:

$$P(A) = \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{x=1}^{t} P(A|B_i, C_k, D_x)\, P(B_i \cap C_k \cap D_x) \quad \text{(A9)}$$

If B_i, C_k, D_x are independent events, Equation A9 becomes:

$$P(A) = \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{x=1}^{t} P(A|B_i, C_k, D_x)\, P(B_i)\, P(C_k)\, P(D_x) \quad \text{(A10)}$$

In applying theTheorem of Total Probability to the calculation of flood probability, the explanations for the terms involved in Equation A10 are as follows:

- P(A) is the unconditional probability of a flood (to be exceeded in any given year);

- $P(A|B_i)$ is the conditional probability of a flood given an input B_i that occurs at the same time as A, not just in the same year;
- $P(B_i)$ is the probability of obtaining a value of B_i for the input B; and
- B, C, D are random variables to the design, for example temporal pattern, losses, storm duration, etc.

A2 JOINT PROBABILITY DISTRIBUTIONS

Jointly distributed random variables: When two or more random variables are considered simultaneously, their joint behaviour is determined by a joint probability law, normally described by a joint cumulative distribution function. When random variables are discrete, a joint probability mass function (PMF), and when they are continuous, a joint probability density (PDF) function is used to describe the governing law of their joint behaviour.

***Joint PDF*:** Consider two random variables X and Y. The probability that X lies between x_1 and x_2 and Y lies between y_1 and y_2 is given by:

$$P[(x_1 \le X \le x_2) \text{ and } (y_1 \le Y \le y_2)] = \int_{x_1}^{x_2} \int_{y_1}^{y_2} p_{X,Y}(x,y)\,dy\,dx \qquad \text{(A11)}$$

This is the volume under the function $p_{X,Y}(x,y)$ over the region. When the region is not a simple rectangle, the integration becomes difficult to evaluate, a common problem encountered in the application of joint probability theory to flood estimation problems.

The joint cumulative distribution function is defined by

$$F_{X,Y}(x,y) \equiv P[(X \le x) \text{ and } (Y \le y)] = \int_{-\infty}^{x} \int_{-\infty}^{y} p_{X,Y}(x,y)\,dy\,dx \qquad \text{(A12)}$$

The density function is a partial derivative of the cumulative function, that is

$$p_{X,Y}(x,y) = \frac{\partial^2}{\partial x\, \partial y} F_{X,Y}(x,y) \qquad \text{(A13)}$$

***Marginal PDF*:** In studying the behaviour of one random variable say X, one may eliminate consideration of the other random variable Y. The behaviour of a particular random variable irrespective of the other is described by the marginal PDF. The marginal PDF of X is obtained by integrating the joint density function over all values of Y:

$$p_X(x) = \int_{-\infty}^{+\infty} p_{X,Y}(x,y)dy \tag{A14}$$

***Conditional PDF*:** If the value of one random variable is known, say Y = y_0, the relative likelihood of the other random variable X taking a value in the interval x, x + dx is $p_{X,Y}(x,y_0)dx$. To obtain a proper density function, these values are normalised by dividing them by their sum:

$$\int_{-\infty}^{+\infty} p_{X,Y}(x,y_0)dx = p_Y(y_0) \tag{A15}$$

The conditional PDF of X when Y is given is defined by:

$$p_{X|Y}(x,y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} \tag{A16}$$

Normally, the marginal distributions are not sufficient to specify the joint distribution. The relationship between the conditional and marginal distribution function determines how much an observation of one variable helps in the prediction of the other (Benjamin and Cornell, 1970).

***Joint PDF of independent random variables*:** If the conditional distribution $p_{X|Y}(x,y)$ is identical to the marginal distribution $p_X(x)$, X and Y are said to be independent random variables. That is when X and Y are independent random variables, their joint probability distribution is the product of their marginal distributions:

$$p_{X,Y}(x,y) = p_X(x)p_Y(y) \tag{A17}$$

The assumption of independence allows one to obtain the joint distribution from only the marginals.

A3 DERIVED DISTRIBUTIONS

In many engineering problems, a functional relationship exists between variables such that the value of one variable (dependent variable) can be predicted given the values of others (independent variables). If, in a probabilistic formulation, the independent variables are considered to be random variables, this randomness is imparted to the dependent variable.

Consider a variable A is dependent on a number of independent random variables B, C, D, . . . according to a functional relationship *f* i.e. $A = f(B, C, D, \ldots)$. The aim of derived distribution theory is to derive the probability distribution of the random variable A given the probability distributions of B, C, D, ... Methods of determining derived distributions are presented in Haan (1977), Walpole and Myers (1993), and Benjamin and Cornell (1970). A summary of the methods can be found in Weinmann (1994).

As described by Sivapalan et al. (1996), the derived flood frequency approach consists of three elements: (i) a statistical model of rainfall, usually expressed in the form of a joint probability distribution of rainfall intensity and duration, including a correction for the effects of catchment size; (ii) a deterministic rainfall-runoff model which contains three components, namely, a runoff generation model, a runoff routing model, and the accounting of antecedent catchment wetness; and (iii) a mathematical framework, or "methodology", within which the above two elements are combined together to permit the "derivation" or estimation of the probability of exceedance of a given flood magnitude, thus, leading to the "derived" flood frequency curve.

A derived probability distribution can be found in two ways: (i) analytical methods and (ii) approximate methods. The choice of a method to compute a derived distribution from these options is influenced mainly by the level of analytical skills and the computer resources available for the task (Weinmann, 1994). These methods are discussed below.

A3.1 ANALYTICAL METHODS

The probability distribution of the dependent variable is found by directly applying principles of probability. In general, the cumulative density function of the dependent variable should be determined. This can be done by enumeration if the probability distributions of the independent variables are discrete, or by integration if the probability distributions of the independent variables are continuous. Then the density function of the dependent variable can be determined by differentiating the cumulative density function.

The main advantage of the analytical approach is that the effects of the various flood producing factors can be clearly distinguished in the final set of equations. However, mathematical complexities in carrying out the required integrations if probability distributions are defined by different functions over different regions often make the method impractical in reality (Benjamin and Cornell, 1970). This approach is only feasible when the rainfall-runoff model and the stochastic models of rainfall and antecedent conditions are simple enough for the derivations to be analytically tractable (Sivapalan et al., 1996).

A3.2 APPROXIMATE METHODS

Discrete methods:

Approximate methods are applied when an analytical approach to the problem becomes difficult or impossible. To simplify calculation procedure, these methods approximate continuous distributions by discrete ones. Derived distributions are then found by enumeration or numerical methods. This means that an approximate solution is obtained by substituting numerical values for variables and parameters involved.

Simulation techniques:

Simulation is often viewed as a "method of last resort", to be employed when everything else has failed (Rubinstein, 1981). It may be defined in very general terms as a technique that involves setting up a model of a real situation and then performing experiments on the model (Naylor et al., 1966). Situations where simulation can be successfully used are well described by Naylor et al. (1966). For example, it can be used to describe the operation of a complex system or to identify important variables and how variables interact.

A special variant of simulation is stochastic simulation, also called Monte Carlo analysis. This is the branch of experimental mathematics concerned with experiments on random numbers (Hammersley and Handscomb, 1964), the latter are essentially independent random variables uniformly distributed over the unit interval (0, 1) (Rubinstein, 1981). Monte Carlo analysis involves performance of a sufficient number of repeated experiments to generate a large number of output values. A histogram of results is then plotted which approximates the desired probability distribution of the dependent variable. Even though the shape of the plotted histogram remains similar, its details will vary as the number of experiments vary. Benjamin and Cornell (1970) conclude that the successful application depends on the appropriateness of the model and the interpretation of the results as much as on the sophistication of the simulation techniques used. Methods are available to account for dependence among variables. Computational efficiency of the simulation process can be enhanced by judicious sampling from the probability distributions (Thompson et al., 1997).

The Cooperative Research Centre for Catchment Hydrology
a cooperative venture formed under the Commonwealth CRC Program between:

Bureau of Meteorology

CSIRO Land and Water

Department of Land and Water Conservation, NSW

Department of Natural Resources and Environment, Vic

Goulburn-Murray Water

Melbourne Water

Monash University

Murray-Darling Basin Commission

Southern Rural Water

The University of Melbourne

Wimmera-Mallee Water

Associates

Department of Natural Resources, Qld

Hydro-Electric Corporation, Tas

State Forests of NSW

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

Centre Office
Department of Civil Engineering, Monash University, Clayton, Victoria 3168, Australia.
Telephone: (03) 9905 2704 Facsimile: (03) 9905 5033 International +61 9905 2704
http://www.catchment.crc.org.au