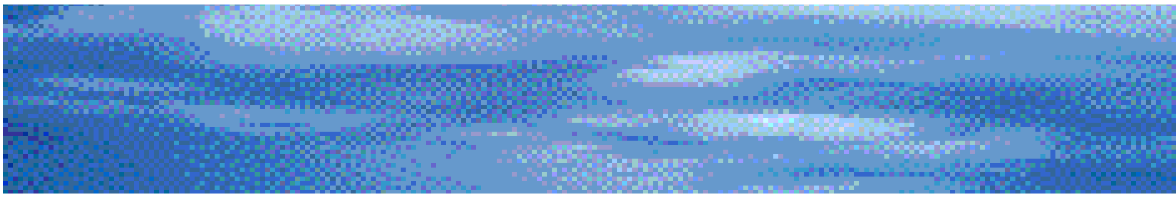
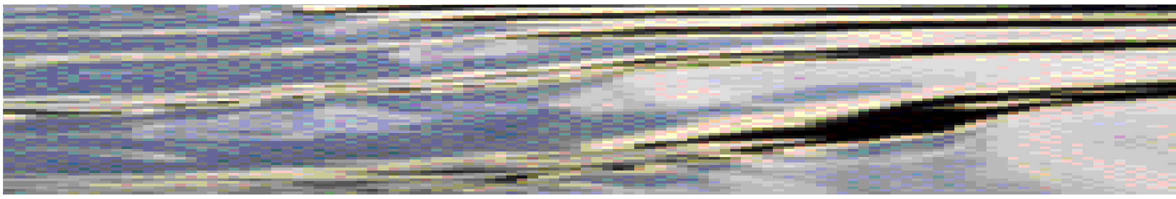
COOPERATIVE RESEARCH CENTRE FOR **CATCHMENT HYDROLOGY**

STOCHASTIC GENERATION OF ANNUAL RAINFALL DATA

TECHNICAL REPORT
Report 02/6
April 2002

Ratnasingham Srikanthan / George Kuczera / Mark Thyer / Tom McMahon





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Bibliography
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1. Rain and rainfall - Australia - Mathematical models. I. Srikanthan, R. (Ratnasingham). II. Cooperative Research Centre for Catchment Hydrology. (Series: Report (Cooperative Research Centre for Catchment Hydrology); 02/6)

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Preface

One of the goals of the Climate Variability Program in the CRC for Catchment Hydrology is to provide catchment and river managers, and other researchers in the CRC, with computer programs to generate climate data. The need is for this at time scales from less than one hour to a year, and for point sites through to large catchments like the Murrumbidgee and the Fitzroy. Our first report (CRC Technical Report 00/16) in this series is a comprehensive literature review; in it a number of techniques are recommended for testing.

This is the first of several reports assessing stochastic data generation techniques. It includes tests of several models to generate stochastically annual rainfall data at 44 sites across Australia.

Summary

This report describes the generation of annual rainfall data with and without incorporating parameter uncertainty. A first order autoregressive [AR(1)] model is generally used to generate annual rainfall data, although recently the use of this model has been criticised because of its inability to model explicitly the wet and dry rainfall years observed in the observed data. To overcome this deficiency, Thyer and Kuczera (2000) developed a hidden state Markov (HSM) model which explicitly models the wet and dry rainfall years.

The HSM model parameters were estimated using the Markov Chain Monte Carlo method; this results in posterior probability distributions for the model parameters. Both the AR(1) and HSM models were applied to 44 rainfall stations located in various parts of Australia and their performance evaluated. When the models were applied without parameter uncertainty, only the HSM model performed satisfactorily for all the stations. However, when parameter uncertainty was incorporated, both models performed equally well, and it was difficult to separate one from the other in terms of model performance.

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1. Introduction

Even though generated annual rainfall data have little direct application, the modelling of annual rainfall data serves two purposes. Firstly, it enables the understanding of the stochastic nature of the annual rainfall data and its implications for long periods of low and high rainfall. This understanding is necessary to manage water supply systems during low rainfall periods. Secondly, stochastic models should be able to maintain their statistical characteristics at different time scales; a good annual rainfall model allows one to disaggregate the generated annual rainfall data into monthly data. In this case, the annual data becomes the input to various disaggregation schemes.

The work reported here forms part of the CRC for Catchment Hydrology (CRCCH) Project 5.2 'National Data Bank of Stochastic Climate and Streamflow Models' in the Climate Variability Program. The review (Srikanthan and McMahon, 2000) carried out as part of the project recommended an autoregressive time series model, or the Hidden State Markov (HSM) model, to generate annual rainfall data. In this report, these two models are applied to 44 rainfall stations located in various parts of Australia. The performance of the models is assessed using a number of basic and other statistics. Based on this, recommendations are made as to the appropriate model for the generation of annual data.

The effect of parameter uncertainty is not considered in the first part of the model comparison. However, the parameter uncertainty is incorporated in the final comparison of the models.

2. Annual Rainfall Data

Forty-four rainfall stations with long records were selected. The locations of the selected rainfall stations are shown in **Figure 1** and the details are given in **Table 1**. Because the strength of persistence in the annual data changed with the starting month of forming the annual totals, the starting months (**Table 1**) were obtained by calibrating the HSM model for annual data formed by different starting months. For more details on this, the reader is referred to Srikanthan et al (2002). The parameters of the annual data are given in **Table 2**.

The data length varies from 69 to 143 years long. The shortest was needed to have at least one rainfall station in each of the CRC for Catchment Hydrology (CRCCH) focus catchments (Tongala, 80056).

The mean annual rainfall of the 44 stations varies from 164 to 1550 mm. The coefficient of variation (C_v) is in the range 0.19 to 0.62. The large values of C_v are associated with low rainfall. Except for two stations (shown in bold in **Table 2**), all the stations have skewness significantly different from zero at 5% level. Only three stations (shown in bold) have lag one autocorrelation coefficient significantly different from zero.

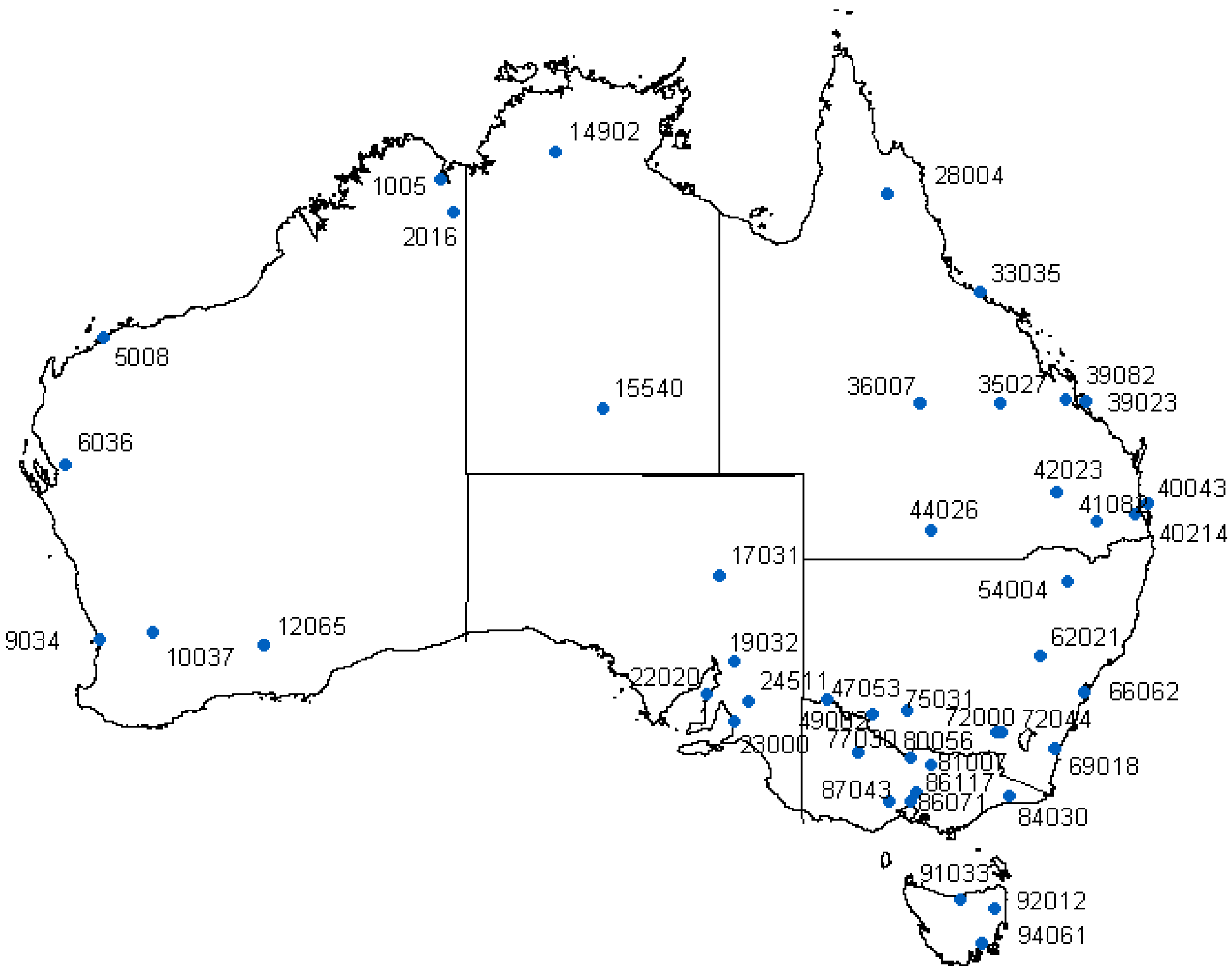


Figure 1 The locations of the selected rainfall stations.

Table 1 The details of the rainfall stations used in the study. The length of record is in years.

No	Number	Name	Latitude	Longitude	Length	Start month
1	5008	Mardie	-21.19	115.98	108	4
2	6036	Meedo	-25.66	114.62	94	4
3	9034	Perth	-34.93	138.58	115	6
4	10037	Cuttening	-31.73	117.76	96	6
5	12065	Norseman Post Office	-32.20	121.78	102	9
6	17031	Marree	-29.65	138.06	113	6
7	19032	Orroroo	-32.74	138.61	118	6
8	22020	Wallaroo	-33.93	137.63	135	12
9	23000	Adelaide	-31.95	115.84	139	6
10	24511	Eudunda	-34.18	139.09	118	8
11	28004	Palmerville	-16.00	144.08	109	9
12	33035	Kalamia Estate	-19.54	147.41	112	4
13	35027	Emerald Post Office	-23.53	148.16	108	8
14	36007	Barcaldine Post Office	-23.55	145.29	112	11
15	39023	Cape Capricorn Lighthouse	-23.48	151.23	87	7
16	39082	Rockhampton Post Office	-23.40	150.50	96	7
17	40043	Cape Moreton Lighthouse	-27.03	153.47	129	7
18	40214	Brisbane	-27.48	153.03	133	7
19	41082	Pittsworth Post Office	-27.71	151.63	112	8
20	42023	Miles Post Office	-26.66	150.18	114	2
21	44026	Cunnamulla Post Office	-28.07	145.68	120	8
22	47053	Wentworth Post Office	-34.11	141.91	132	1
23	49002	Balranald RSL	-34.64	143.56	121	11
24	54004	Bingara Post Office	-29.87	150.57	113	8
25	62021	Mudgee (George Street)	-32.59	149.58	122	7
26	66062	Sydney	-33.86	151.20	140	8
27	69018	Moruya Heads Pilot Station	-35.91	150.15	123	5
28	72000	Adelong	-35.31	148.06	115	8
29	72044	Tumut	-35.30	148.22	113	8
30	75031	Hay Miller Street	-34.52	144.85	119	1
31	77030	Narraport	-36.01	143.03	112	8
32	80056	Tongala	-36.25	144.95	69	8
33	81007	Caniambo	-36.46	145.66	95	8
34	84030	Orbost	-37.63	148.46	115	8
35	86071	Melbourne	-37.81	144.97	143	6
36	86117	Toorourrong Reservoir	-37.48	145.15	106	9
37	87043	Meredith (Darra)	-37.82	144.15	124	6
38	91033	Frankford (Rossville)	-41.32	146.73	106	12
39	92012	Fingal (Forestry Legge Street)	-41.64	147.97	110	8
40	94061	Sandford (Maydena)	-42.93	147.52	111	12
41	1005	Wyndham Port	-15.46	128.10	79	3
42	2016	Lissadell	-16.67	128.57	105	1
43	14902	Katherine Council	-14.46	132.26	111	4
44	15540	Alice Springs Post Office	-23.71	133.87	112	2

Table 2 The parameters of the annual data.

Number	Name	Mean (mm)	C_v	Skew	Lag one autocorrel
5008	Mardie	276	0.62	0.79	-0.11
6036	Meedo	216	0.42	0.88	**0.24**
9034	Perth	868	0.18	0.72	-0.02
10037	Cuttening	312	0.24	0.51	0.06
12065	Norseman Post Office	287	0.32	1.24	0.01
17031	Marree	164	0.51	1.53	0.06
19032	Orroroo	341	0.32	1.44	0.02
22020	Wallaroo	360	0.23	**0.29**	-0.05
23000	Adelaide	530	0.20	0.63	0.09
24511	Eudunda	446	0.25	1.21	0.07
28004	Palmerville	1034	0.31	0.62	0.07
33035	Kalamia Estate	1085	0.46	0.85	0.06
35027	Emerald Post Office	642	0.33	0.75	**0.22**
36007	Barcaldine Post Office	496	0.43	0.88	0.17
39023	Cape Capricorn Lighthouse	801	0.33	0.69	0.09
39082	Rockhampton Post Office	946	0.38	0.97	0.09
40043	Cape Moreton Lighthouse	1550	0.26	0.95	0.00
40214	Brisbane	1154	0.32	1.01	0.05
41082	Pittsworth Post Office	703	0.23	0.86	-0.12
42023	Miles Post Office	661	0.32	0.51	-0.12
44026	Cunnamulla Post Office	374	0.42	1.25	0.10
47053	Wentworth Post Office	288	0.35	1.17	0.16
49002	Balranald RSL	322	0.34	1.00	0.18
54004	Bingara Post Office	745	0.27	0.54	0.04
62021	Mudgee (George Street)	670	0.26	0.79	0.09
66062	Sydney	1226	0.27	0.73	0.16
69018	Moruya Heads Pilot Station	972	0.32	0.71	**0.27**
72000	Adelong	795	0.24	0.72	0.08
72044	Tumut	822	0.23	0.67	0.05
75031	Hay Miller Street	369	0.34	1.22	0.16
77030	Narraport	354	0.29	0.48	0.17
80056	Tongala	443	0.28	1.56	0.23
81007	Caniambo	524	0.27	0.99	0.06
84030	Orbost	855	0.24	0.74	-0.16
86071	Melbourne	657	0.19	0.45	0.10
86117	Toorourrong Reservoir	804	0.20	**0.43**	0.03
87043	Meredith (Darra)	685	0.19	0.36	0.09
91033	Frankford (Rossville)	1069	0.22	0.52	0.01
92012	Fingal (Forestry Legge Street)	611	0.27	1.29	0.11
94061	Sandford (Maydena)	578	0.22	0.63	0.10
1005	Wyndham Port	695	0.32	0.78	0.06
2016	Lissadell	616	0.30	0.89	0.14
14902	Katherine Council	974	0.28	0.61	0.04
15540	Alice Springs Post Office	280	0.50	1.48	**0.28**

3. Model Evaluation

The following parameters are used to evaluate the data generation model for annual rainfall:

- Mean ($\bar{x}$)
- Standard deviation (s)
- Coefficient of skewness (g)
- Lag one autocorrelation coefficient (r)
- Maximum rainfall
- Minimum rainfall
- Adjusted range (R)
- 2, 3, 5, 7 and 10-year low rainfall totals

For convenience, the last four items in the above list are standardised by dividing by the historical mean annual rainfall. The first four items are usually estimated from the following expressions.

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \qquad (1)$$

$$s = \sqrt{\frac{1}{(n-1)}\sum_{t=1}^{n}(x_t - \bar{x})^2} \qquad (2)$$

$$g = \frac{n}{(n-1)(n-2)s^3}\sum_{t=1}^{n}(x_t - \bar{x})^3 \qquad (3)$$

$$r = \frac{1}{(n-1)s^2}\sum_{t=1}^{n-1}(x_{t+1} - \bar{x})(x_t - \bar{x}) \qquad (4)$$

In the above equations, x_t represents the annual rainfall for year t and n is the number of years of data. Stedinger and Taylor (1982) suggest the use of the population mean (i.e. the mean estimated from the historic data) rather than the generated mean to overcome the bias in the standard deviation, skewness and correlation. The adjusted range (R) is obtained from

$$R = \max\{D_k\} - \min\{D_k\} \qquad k = 1, 2, \dots, n$$

where $$D_k = \sum_{t=1}^{k}(x_t - \bar{x}) \qquad (5)$$

The maximum, minimum rainfall, adjusted range and low rainfall sums are standardised by dividing them by the mean annual rainfall.

4. First Order Autoregressive Model

Rainfall data are less variable than streamflow data, with little correlation between the values in successive years. Hence, a first order autoregressive [AR(1)] or a random model is adequate for most cases. The AR(1) model is of the form:

$$X_t = rX_{t-1} + (1 - r^2)^{1/2}\,\eta_t \tag{6}$$

where X_t standardised rainfall in year t,
η_t normally distributed random component with zero mean and unit variance, and
r lag one autocorrelation coefficient.

The annual rainfall amount is obtained from

$$x_t = \bar{x} + s\,X_t \tag{7}$$

If the annual data are skewed, the skewness in the data can be modelled through the Wilson-Hilferty transformation:

$$\varepsilon_t = \frac{2}{g_\varepsilon}\left\{\left(1 + \frac{g_\varepsilon \eta_t}{6} - \frac{g_\varepsilon^2}{36}\right)^3 - 1\right\} \tag{8}$$

where g_ε is the skewness of ε_t which is related to the skewness (g) of annual data through

$$g_\varepsilon = \frac{(1 - r^3)}{(1 - r^2)^{3/2}}\,g \tag{9}$$

Since only two stations have skewness which is not significantly different from zero, the Wilson-Hilferty transformation is used for all the stations. Most of the stations have small lag one autocorrelation coefficients. An AR(1) model is used for the stations with lag one autocorrelation coefficient greater than 0.05 and a random model for the other stations.

4.1 Parameter Uncertainty for Normally Distributed Data

Stedinger and Taylor (1982) derived expressions for incorporating parameter uncertainty in the AR(1) model for normally distributed data. Following their notation, normally distributed annual rainfall can be obtained from

$$x_t = \alpha + \beta x_{t-1} + e_t \tag{10}$$

The unknown model parameters are α, β and σ_ε^2 (variance of e_t). These parameters are related to the mean, variance and lag one autocorrelation coefficient through:

$$\alpha = (1 - r)\,\bar{x}\,;\ \beta = r \ \text{ and } \sigma_\varepsilon^2 = (1 - r^2)s^2$$

α and β can be obtained by minimising

$$L = \sum_{t=2}^{n}(x_t - \alpha - \beta x_{t-1})^2$$

This is equivalent to minimising $(y - X\theta)^T(y - X\theta)$ where $y = (x_2, \ldots, x_n)^T$, $\theta = (\alpha, \beta)^T$ and

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ . & . \\ 1 & x_{n-1} \end{bmatrix}$$

The solution to θ is given by $\hat{\theta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$. The variance of the residuals is estimated from:

$$\begin{aligned} s_e^2 &= \frac{1}{n-3}\sum_{t=2}^{n}(x_t - \hat{\alpha} - \hat{\beta}x_{t-1})^2 \\ &= \frac{1}{n-3}(y - X\theta)^T(y - X\theta) \end{aligned} \tag{11}$$

Annual rainfall data is generated by sampling σ_ε^2 from a Chi squared distribution with (n-3) degrees of freedom and θ from a Normal distribution with mean $\hat{\theta}$ and variance $\hat{\sigma}_\varepsilon^2\,(\mathbf{X}^T\mathbf{X})^{-1}$.

$$\hat{\sigma}_\varepsilon^2 = \frac{(n-3)s_e^2}{\chi^2} \tag{12}$$

where χ^2 is a random Chi squared variable with (n-3) degrees of freedom.

4.2 Parameter Uncertainty for Non-normally Distributed Data

Thyer and Kuczera (2001) developed a procedure for fully evaluating the uncertainty of the parameters in the AR(1) model when used with the Box-Cox transformation to normalise the data. The AR(1) model can be rewritten in the form:

$$z_t = \mu + \phi_1(z_{t-1} - \mu) + \varepsilon_t \tag{13}$$

where z_t is a normally distributed time series, μ is the mean, ϕ_1 is the autoregressive parameter and is a Gaussian random variable with zero mean and variance σ_ε^2.

In the case of non-normal data, the data is normalised using the Box-Cox transformation (Box and Cox, 1964):

$$z_t = \begin{cases} \dfrac{x_t^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log x_t & \lambda = 0 \end{cases} \qquad (14)$$

where λ is the transformation parameter which transforms the skewed data to approximately Gaussian. With the transformation parameter included, the vector of unknown AR(1) model parameters is given by:

$$\boldsymbol{\theta}^T = (\mu, \sigma_\varepsilon, \phi_1, \lambda) \qquad (15)$$

The Metropolis algorithm is used to simulate the posterior distribution of the model parameters. To improve the acceptance rate of the Metropolis algorithm, the parameter space is transformed to remove the dependencies of μ and σ_ε on λ.

$$\mu = \frac{\mu_x^\lambda - 1}{\lambda}$$
$$\sigma_\varepsilon = \mu_x^{\lambda-1}\sigma_x\sqrt{1-\phi_1^2} \qquad (16)$$

where μ_x and σ_x represent a first order approximation to the expected value and the standard deviation of the untransformed rainfall data.

The expected value and the standard deviation of the AR(1) model parameters are given in **Table 3**.

Table 3 Expected values and the standard deviation of the AR(1) model parameters using the Box-Cox transformation.

	Mean		Standard deviation		Box-Cox λ		Lag 1 autocorrel	
Station	**Exp value**	**Std dev**	**Exp value**	**Std dev**	**Exp value**	**Std dev**	**Exp value**	**Std dev**
Wyndham	661	30	210	23	-0.213	0.353	0.122	0.118
Lissadell	601	22	182	14	0.349	0.267	0.140	0.105
Mardie	62	52	1099	663	1.369	0.119	-0.572	0.266
Meedo	204	14	89	9.2	0.317	0.239	0.270	0.108
Perth	852	16	155	11	-0.207	0.427	-0.002	0.096
Cuttening	305	9.4	76	6.2	0.042	0.393	0.101	0.104
Norseman	273	9.3	84	7.1	-0.111	0.255	0.040	0.100
Katherine	969	33	266	31	1.004	0.540	0.098	0.109
Alice Springs	253	16	124	12	0.078	0.159	0.260	0.096
Marree	148	8.6	77	6.6	0.188	0.144	0.085	0.099
Orroroo	323	11	100	8.1	-0.173	0.233	0.104	0.093
Wallaroo	353	7.8	84	5.4	0.232	0.382	-0.058	0.092
Adelaide	520	11	104	6.8	0.039	0.344	0.104	0.086
Eudunda	429	12	105	8.3	-0.391	0.306	0.122	0.097
Palmerville	1001	36	316	23	0.284	0.266	0.075	0.095
Kalamia	991	53	470	41	0.091	0.195	0.044	0.098
Emerald	619	28	212	18	0.297	0.230	0.239	0.097
Barcaldine	465	25	207	17	0.262	0.179	0.160	0.098
Capricorn	765	36	262	25	0.075	0.294	0.135	0.110
Rockhampton	867	39	315	38	-0.457	0.344	0.066	0.104
Cape Moreton	1496	38	375	27	-0.198	0.289	0.068	0.090
Brisbane	1105	36	353	25	0.063	0.218	0.072	0.091
Pittsworth	683	15	157	12	-0.148	0.349	-0.099	0.097
Miles	634	20	212	16	0.204	0.273	-0.131	0.096
Cunnamulla	348	16	145	12	0.068	0.187	0.106	0.093
Wentworth	277	10	97	6.8	0.291	0.185	0.125	0.088
Balranald	312	13	107	8.0	0.353	0.200	0.148	0.093
Bingara	722	22	199	15	0.107	0.334	0.039	0.098
Mudgee	648	19	168	13	-0.090	0.307	0.106	0.092
Sydney	1186	35	325	23	0.042	0.281	0.143	0.085
Moruya	924	40	299	28	-0.147	0.279	0.284	0.090
Adelong	778	22	190	14	0.170	0.291	0.130	0.095
Tumut	804	22	191	14	0.117	0.313	0.105	0.097
Hay	353	14	121	9.3	0.156	0.199	0.150	0.095
Narraport	345	13	104	8.3	0.324	0.271	0.209	0.095
Tongala	427	20	114	15	-0.373	0.349	0.311	0.127
Caniambo	506	16	133	12	-0.167	0.330	0.044	0.106
Orbost	829	18	194	14	-0.179	0.355	-0.144	0.092
Melbourne	646	13	122	8.0	0.079	0.390	0.120	0.084
Toorourrong	791	19	160	12	0.128	0.427	0.067	0.098
Meredith	674	15	131	9.4	0.082	0.432	0.113	0.091
Frankford	1042	25	228	18	-0.173	0.427	-0.011	0.099
Fingal	580	18	146	14	-0.724	0.326	0.154	0.099
Sandford	564	14	123	9.6	-0.168	0.382	0.110	0.095

5. Hidden State Markov Model

The HSM model (**Figure 2**) assumes the climate is in one of two states: wet (W) or dry (D). Each state has an independent rainfall distribution, assumed to be Gaussian. The time spent in each state is governed by the state transition probabilities. This provides an explicit mechanism to replicate the variable length of wet and dry cycles.

The simulation of annual rainfall time series is a two-step process. In the first step the climate state at year *t*, s_t, is simulated by a Markovian process:

$$s_t \mid s_{t-1} \sim \text{Markov}(\boldsymbol{P}) \tag{17}$$

where $\boldsymbol{P}$ is a (2x2) state transition probability matrix whose elements are:

$$p_{ij} = Pr(s_t = j \mid s_{t-1} = i) \quad i, j = W, D \tag{18}$$

Once the state for year *t* is known, the rainfall is simulated using:

$$y_t \sim \begin{cases} \text{N}(\mu_W, \sigma_W^2) & \text{if } s_t = W \\ \text{N}(\mu_D, \sigma_D^2) & \text{if } s_t = D \end{cases} \tag{19}$$

where $\text{N}(\mu,\sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2. Therefore, the vector of unknown parameters for the HSM model, θ, is composed of the rainfall distribution parameters for each state, the transition probabilities, and the hidden state time series, $S_N = \{s_1, s_2, \ldots s_N\}$ where:

$$\boldsymbol{\theta}' = (\mu_W, \sigma_W, \mu_D, \sigma_D, \boldsymbol{P}, S_N) \tag{20}$$

Prior to model calibration the hidden state time series is unknown. Thus it is included as a model parameter to be estimated during the calibration process.

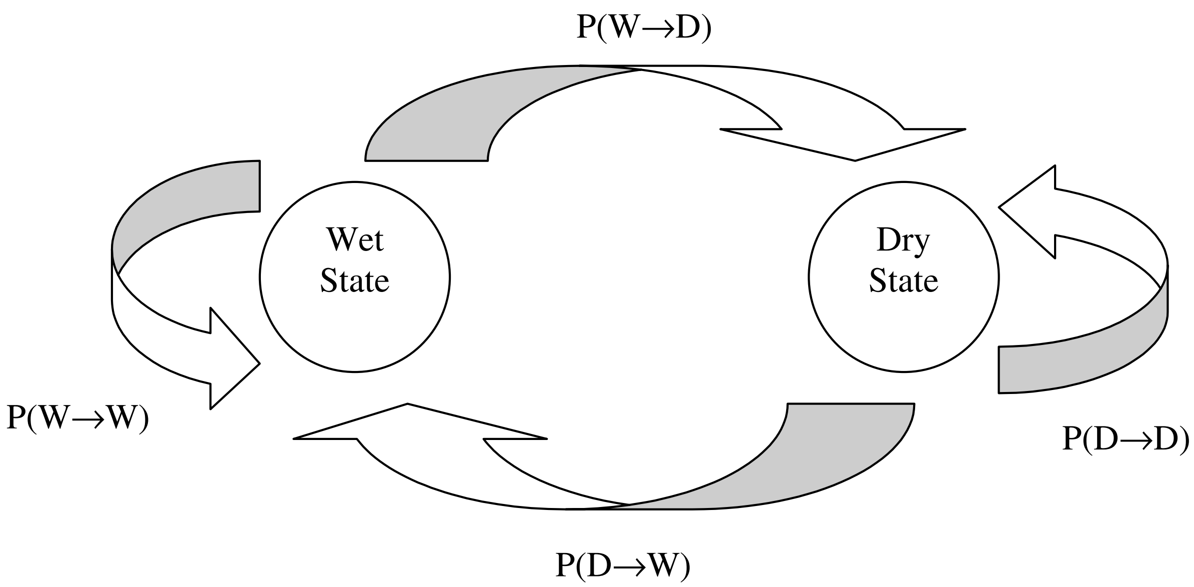


Figure 2 Schematic representation of the HSM model.

5.1 Model Calibration

For model calibration a Bayesian framework is used to infer the distribution of the model parameters, θ, for the given time series data, Y_N. This distribution is referred to as the posterior distribution of the model parameters, $p(\theta \backslash Y_N)$. For the HSM model it is not possible to derive an analytical expression for the posterior distribution. Thus, Markov Chain Monte Carlo (MCMC) simulation methods are employed to draw samples from the posterior distribution. The basic idea of MCMC methods is to simulate a Markov chain iterative sequence where, at each iteration, a sample of the model parameters, θ, is generated. Given certain conditions the distribution of these samples converges to a stationary distribution which is the posterior distribution, $p(\theta \backslash Y_N)$. To calibrate the HSM model, the MCMC method known as the Gibbs sampler is applied. The details of the calibration process are given in Thyer and Kuczera (2000).

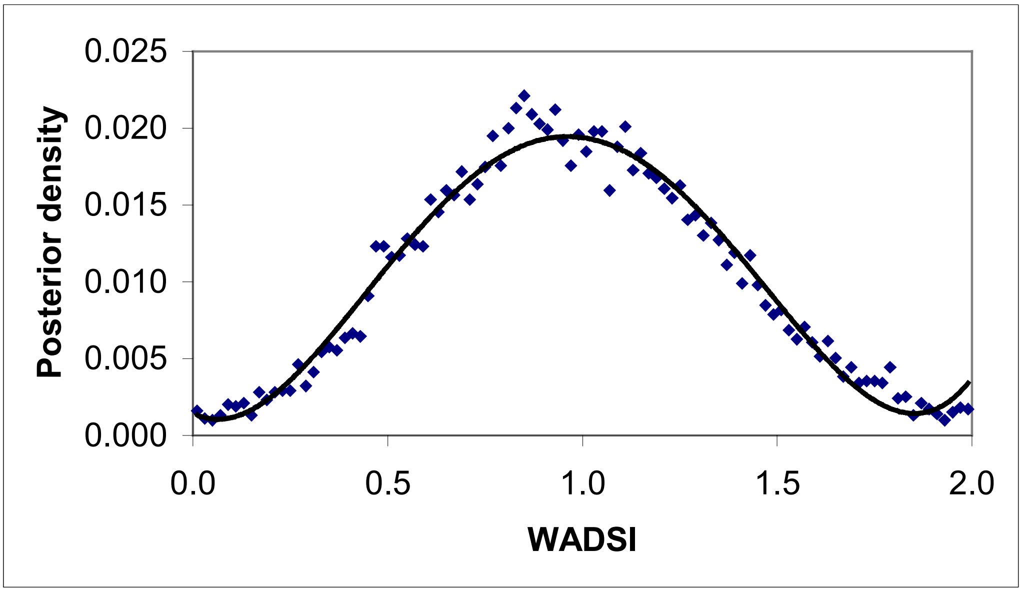
5.2 Application of HSM Model

The HSM model was calibrated for the 44 Australian rainfall stations using twelve different months to form the annual rainfall totals. Several indices are used to interpret the results and these are briefly defined below.

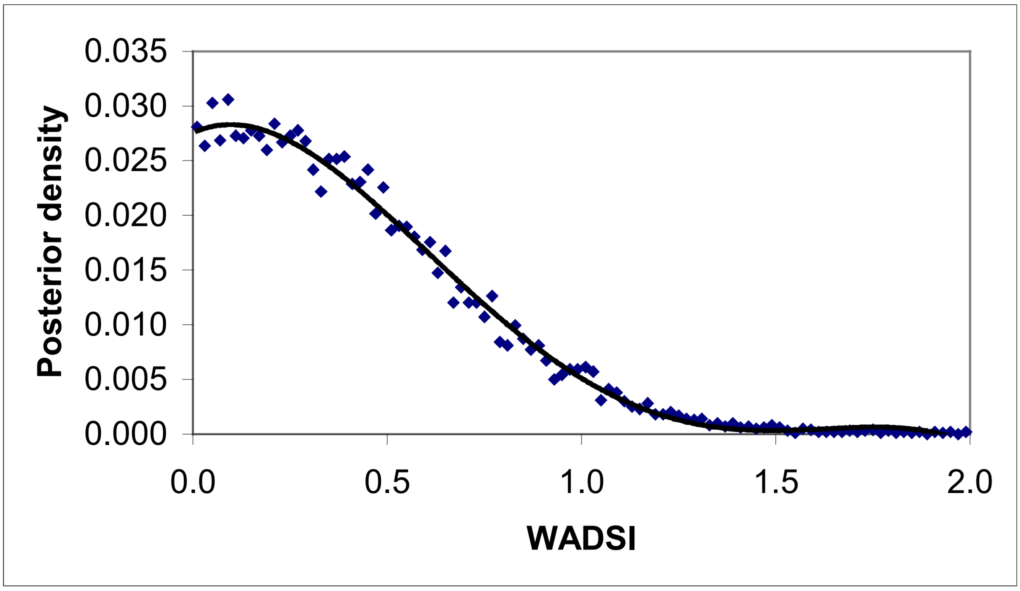
The wet and dry separation index (*WADSI*) is defined as:

$$WADSI = \frac{\mu_W - \mu_D}{\sqrt{\left(\sigma_W^2 + \sigma_D^2\right)}} \tag{21}$$

This index is a convenient measure of the separation between the wet and dry states. If the difference between the wet and dry means is large, then the value of *WADSI* will be relatively high. As the value of *WADSI* increases, the wet and dry distributions become more separated and easier to identify. A plot of the distribution of *WADSI* is used to identify sites that have separate wet and dry distributions. The distributions of *WADSI* for Meedo (presence of two states) and Perth (absence of two states) are shown in **Figure 3**.



(a) Meedo showing the existence of two separate states.



(b) Perth showing the non-existence of two states.

Figure 3 Plots of the distribution of the wet and dry separation index (WADSI).

The state stability index (*SSI*) is defined as follows:

$$SSI = \frac{\sum |P(W) - 0.5|}{N} \qquad (22)$$

where P(W) is the probability of wet state and *N* the number of years of data.

Values of *SSI* close to zero indicate no persistence in the rainfall to stay in either wet or dry state. Values of *SSI* around 0.3 generally indicate persistence, but this needs to be confirmed with a visual inspection of a time series plot of the posterior probability of a year being classified as wet $\{P(s_t = W \backslash Y_N)$. A plot of $P((s_t = W \backslash Y_N)$ versus year for Wyndham is shown in **Figure 4**.

The strength of persistence structure is assessed using the expected state resident times (SRT) and is obtained as the reciprocal of the transition probabilities.

$$E(SRT_D) = \frac{1}{p_{DW}} \qquad E(SRT_W) = \frac{1}{p_{WD}} \qquad (23)$$

The strength of persistence structure (*SPS*) was classified from the state residence times using **Table 4** (Thyer, 2001).

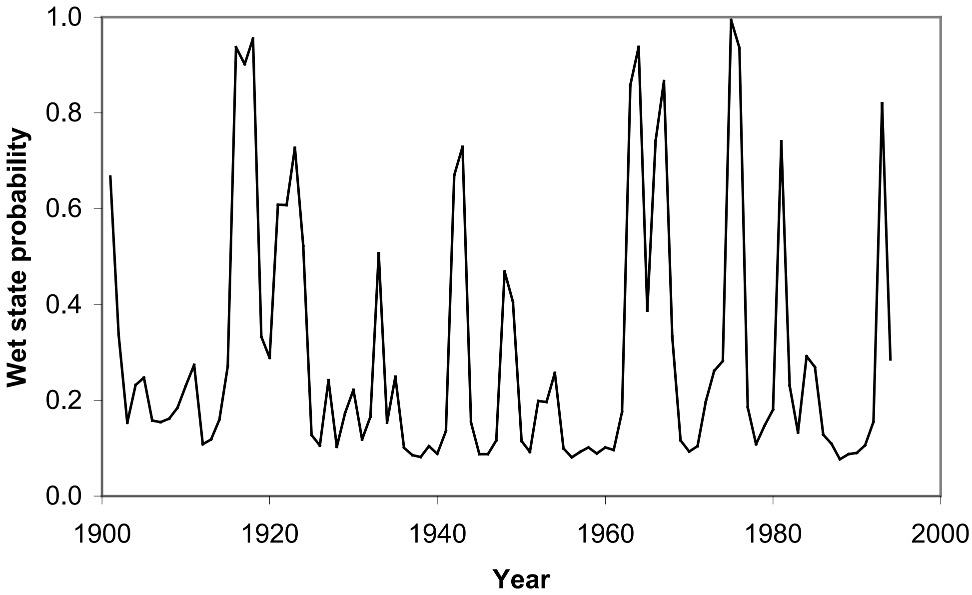


Figure 4 Time series plot of the posterior probability of a year being classified as wet for Wyndham.

Table 4 The classification of the strength of persistence structure.

Expected residence time (years)	Classification
1 - 4	Weak (W)
4 - 10	Medium (M)
10 - 25	Strong (S)
> 25	Very Strong (VS)

The two-state persistence structure is denoted by [SPS_W,SPS_D,WADSI] where the subscripts denote the wet and dry states. The full set of the calibration results are given in Srikanthan et al (2002a). Of the 44 stations used, 12 are highly unlikely to have a two-state persistence structure (**Figure 5**). Of the remaining 32 stations, 15 are highly likely to have, and 17 stations possibly have, a two-state persistence structure. These 32 stations with the chosen starting months and the two-state persistence structure are given in **Table 5**.

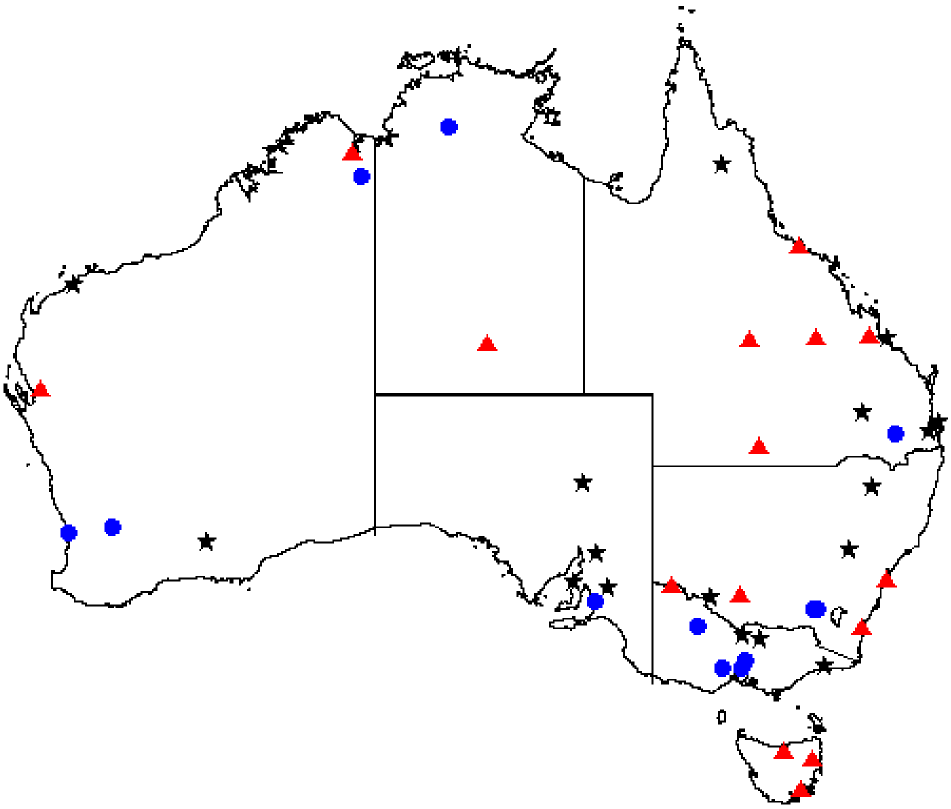


Figure 5 The locations of the stations that are highly unlikely to have two-state persistence (circles), highly likely to have two-state persistence (triangles) and possibly have two-state persistence (stars).

Table 5 The rainfall stations indicating highly likely and possibly to have two-state persistence structure.

Number	Name	Starting month	Two state persistence structure
Stations highly likely to have two-state persistence			
006036	Meedo	4	[W, M, 0.991]
033035	Kalamia Estate	4	[W, M, 1.173]
035027	Emerald Post Office	8	[W, M, 0.924]
036007	Barcaldine Post Office	11	[W, M, 1.176]
039082	Rockhampton Post Office	7	[W, M, 1.183]
044026	Cunnamulla Post Office	8	[W, M, 1.183]
047053	Wentworth Post office	1	[W, S, 0.922]
066062	Sydney	8	[W, M, 0.993]
069018	Moruya Heads Pilot Station	5	[W, M, 1.323]
075031	Hay Miller Street	1	[W, M, 0.926]
091033	Frankford (Rossville)	12	[W, M, 0.910]
092012	Fingal (Forestry Legge Street)	8	[W, M, 0.986]
094061	Sandford (Maydena)	12	[W, M, 1.429]
001005	Wyndham	3	[W, W, 1.203]
015540	Alice Springs	1	[W, S, 1.236]
Stations possibly to have two-state persistence			
005008	Mardie	4	[W, M, 0.970]
012065	Norseman Post Office	9	[W, M, 0.834]
017031	Marree	6	[W, M, 1.116]
019032	Orroroo	6	[W, M, 0.794]
022020	Wallaroo	12	[W, M, 1.014]
024511	Eudunda	8	[W, M, 0.822]
028004	Palmerville	9	[W, M, 0.724]
039023	Cape Capricorn	7	[W, M, 0.839]
040043	Cape Moreton	7	[W, W, 0.842]
040214	Brisbane	7	[W, M, 0.890]
054004	Bingara	8	[W, M, 0.826]
062021	Mudgee	7	[W, W, 0.764]
042023	Miles Post Office	2	[W, M, 1.013]
049002	Balranald RSL	11	[W, M, 0.938]
080056	Tongala	8	[W, M, 0.940]
081007	Caniambo	8	[W, M, 0.842]
084030	Orbost	8	[W, W, 0.808]

The estimated parameters of the HSM model are given in Table 6. These are the expected values and are used to generate data without parameter uncertainty.

Table 6 The estimated parameters for the HSM model.

Number	p_{WD}	p_{DW}	μ_W (mm)	μ_D (mm)	σ_W (mm)	σ_D (mm)
001005	0.55	0.38	874	579	211	125
005008	0.76	0.62	389	191	178	101
006036	0.48	0.21	297	185	98	63
012065	0.62	0.22	372	261	119	59
015540	0.57	0.15	473	235	175	82
017031	0.75	0.17	267	144	122	56
019032	0.81	0.24	453	315	157	76
022020	0.46	0.58	405	309	75	57
024511	0.75	0.21	566	420	157	82
028004	0.38	0.25	1228	943	369	244
033035	0.53	0.30	1524	844	503	291
035027	0.46	0.23	822	568	237	140
036007	0.57	0.32	693	398	219	123
039023	0.42	0.36	957	680	280	175
039082	0.52	0.32	1255	765	365	196
040043	0.72	0.39	1819	1430	478	278
040214	0.66	0.20	1515	1059	469	267
042023	0.57	0.58	791	542	204	136
044026	0.66	0.14	593	330	198	102
047053	0.55	0.10	418	268	145	72
049002	0.53	0.10	475	305	160	85
054002	0.64	0.26	896	697	225	151
062021	0.49	0.30	787	609	195	124
066062	0.43	0.16	1539	1112	363	232
069018	0.31	0.21	1236	792	285	177
075031	0.69	0.14	537	344	186	95
080056	0.65	0.14	606	416	186	80
081007	0.61	0.25	653	486	174	96
084030	0.77	0.50	985	781	219	147
091033	0.50	0.48	1205	954	227	158
092012	0.60	0.25	772	549	200	104
094061	0.48	0.32	691	507	111	65

6. Discussion of Results

6.1 Without Parameter Uncertainty

One hundred replicates each of length equal to the historical data length were generated by using constant parameter values estimated from the historical record. The average of the parameters from the 100 replicates and the 2.5%, 25%, 50%, 75% and 97.5% values were obtained. Full sets of results can be found in Srikanthan et al. (2002b). Both the models preserved the basic parameters, adjusted range and the extreme rainfalls. However, the AR(1) model did not preserve the 2- and 3-year low rainfall sums. The number of historical minimum 2-, 3-, 5-, 7- and 10-year rainfall sums that were outside the 95 % confidence limits are given in **Table 7**.

Using the Binomial distribution, the number of stations expected to fall outside the 95% confidence limits is about 4 for 44 stations. Hence, the AR(1) model is not consistent with the data. Although one may question the spatial independence assumption, the prima facia evidence is that the AR(1) N_{obs} values for the 2- and 3-year sums are unusually high. **Table 7** shows that, if the AR(1) model were the correct model, the probability of having 10 or more stations outside the 95% confidence limits for a total of 44 stations is less than 1 chance in 100,000. This suggests strongly that the AR(1) model is unable to simulate 2- and 3-year rainfall sums.

6.2 With Parameter Uncertainty

The calibration of the models using MCMC resulted in a set of 10,000 parameters for HSM model and 20,000 for the AR(1) model. One thousand replicates of data were generated by picking 1000 sets of parameters at random and various model evaluation statistics were estimated. In the case of Mardie, a large number of very high values were generated using the AR(1) model with Box-Cox transformation; this generated data was not used in the model comparisons. To model this adequately, another distribution would need to be adopted. The averages of the statistics from the 1000 replicates along with the historical values are presented in **Table A1 - A12** in **Appendix A**. The ranges of the parameters from the 1000 replicates are shown in **Figures A1 to A12**.

The mean annual rainfall was well reproduced by both the models (**Table A1**); none of the means was outside the 95 % confidence limits (**Figure A1**). The annual standard deviation was also well reproduced by both the models (**Table A2**). The variation in the values for standard deviation estimated from the replicates is shown in **Figure A2**. The historical standard deviation is within the 95 % confidence limits for all the stations. The skewness was preserved satisfactorily by both the models (**Table A3**). Except for a few cases for the AR(1) and HSM models, the lag one autocorrelation coefficient of the annual rainfall was satisfactorily reproduced (**Table A4**).

The maximum annual rainfall was generated satisfactorily by both the models (**Table A5**). **Figure**

Table 7 The number of historical minimum 2-, 3-, 5-, 7- and 10-year rainfall sums that were outside the 95% confidence limits.

Model	Number of stations	Duration (years)	N_{obs}	Binomial $P(N \le N_{obs})$
AR(1)	44	2	9	1.000
		3	12	1.000
		5	0	0.10
		7	0	0.10
		10	0	0.10
HSM	32	2	0	0.19
		3	0	0.19
		5	0	0.19
		7	0	0.19
		10	0	0.19

A5 shows that all the historical maximum values were within the 95 % confidence limits (**Figure A5**). The minimum annual rainfall was satisfactorily generated (**Table A6**) and all the historical minimum values were also within the 95 % confidence limits.

For all the sites except Mardie, Perth, Katherine, Cape Capricorn, Brisbane, Cunnamulla and Moruya, the average value of the adjusted range was close to the corresponding values (**Table A7**). However, the observed values were within the 95 % confidence limits for all the sites (**Figure A7**).

The minimum 2-, 3-, 5-, 7- and 10-year rainfall sums were preserved for all the sites by both the models (**Figures A8 - A12**). AR(1) performed better with parameter uncertainty incorporated than without it.

The autocorrelation function calculated from the generated data using AR(1) and HSM models for Wyndham and Sydney are shown in **Figures A13** and **A14** respectively. There is no noticeable difference between the autocorrelation functions of both the models.

7. Conclusions

The lag one Markov or AR(1) model and the HSM model were evaluated with and without parameter uncertainty using annual rainfall data from 44 stations located in various parts of Australia. For the HSM model, it is highly likely, or possible to have, a two-state persistence structure for 32 stations. One hundred replicates, each of length equal to the historical record, were generated and several model evaluation statistics computed. The AR(1) model was found not to be adequate for all the stations when applied without parameter uncertainty. The HSM model was found to be adequate for all the stations when applied without parameter uncertainty.

When parameter uncertainty was incorporated in the data generation procedure, the observed statistics cannot discriminate between AR(1) and HSM (Readers should recall that the AR(1) model was not used to generate data for Mardie). Both models performed equally well.

8. References

Box, G. E. and Cox, D. R., 1964. An analysis of transformations. Jour. Royal Stat. Soc., Ser. B, 26(2), 211-252.

Srikanthan, R. and McMahon, T. A., 2000. Stochastic generation climate data: A review. CRCCH Report 00/16, Monash University, Clayton, 34pp.

Srikanthan, R., Thyer, M.A., Kuczera, G. A and McMahon, T. A, 2002a. Application of hidden state Markov model to Australian annual rainfall data. CRC for Catchment Hydrology Working Document 02/4.

Srikanthan, R., Pegram, G.G.S., Thyer, M.A., Kuczera, G. A., and McMahon, T. A 2002b. Generation of annual rainfall data for Australian stations. CRC for Catchment Hydrology Working Document 02/3.

Stedinger, J. R. and Taylor, M. R., 1982. Synthetic streamflow generation: 2. Effect of parameter uncertainty. Water Resources Research, 18(4): 919-924.

Thyer, M. A., 2001. Modelling long-term persistence in hydrological time series. Ph. D. Thesis, University of Newcastle.

Thyer, M. A. and Kuczera, G. A., 2000. Modelling long-term persistence in hydro-climatic time series using a hidden state Markov model. Water Resources Research, 36(11): 3301-3310.

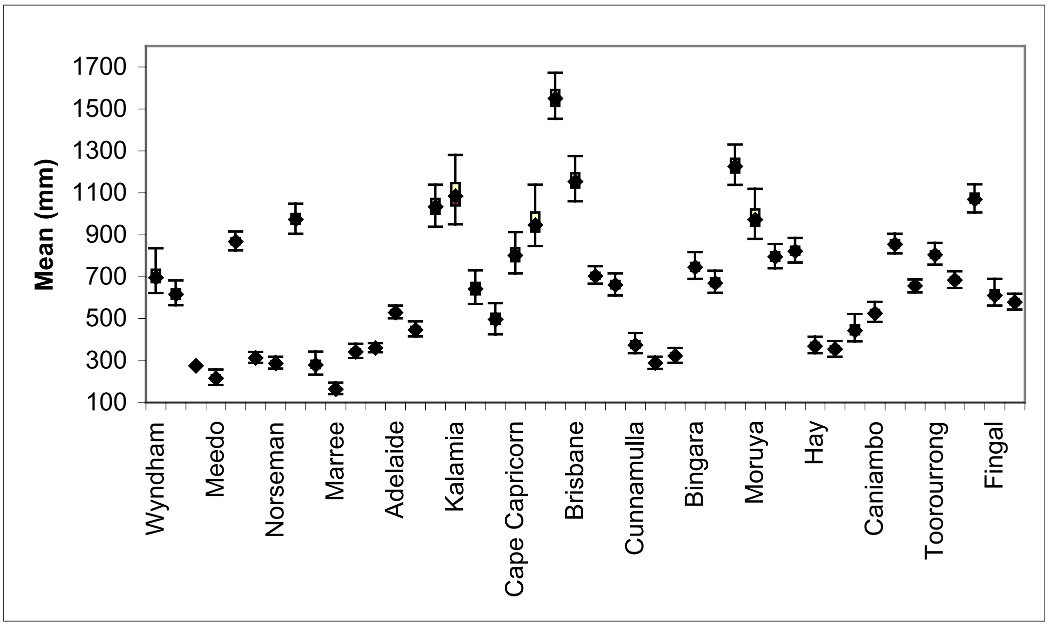
Thyer, M. A. and Kuczera, G., 2001. Quantifying parameter uncertainty in stochastic models using the Box-Cox transformation. Submitted to Jour. Hydrol.

Appendix A Comparison of Historical and Generated Parameters

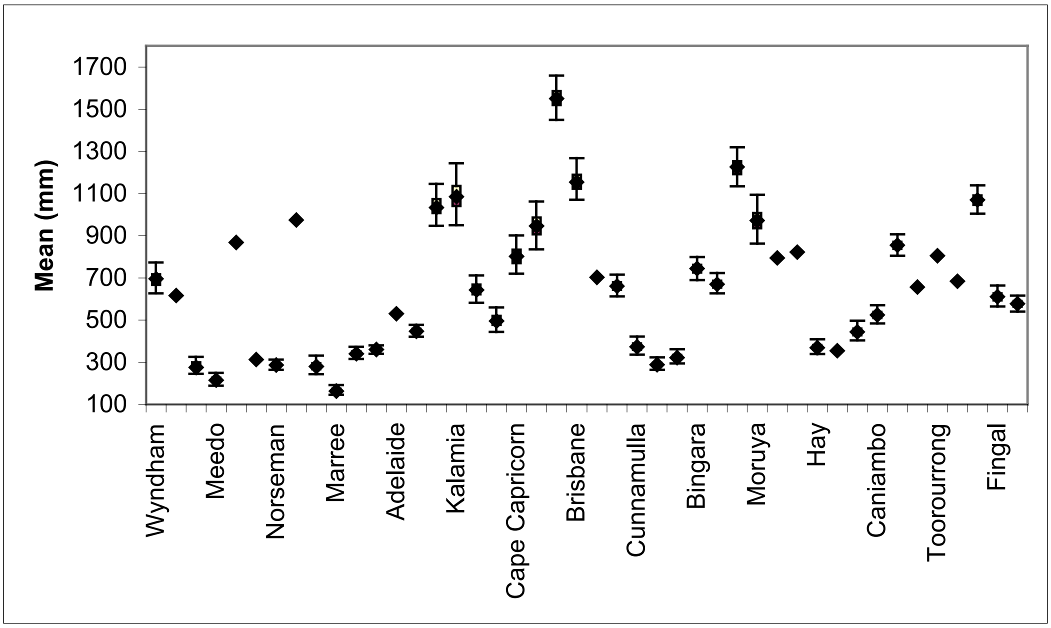
The results presented here are from the model runs with parameter uncertainty. The historical parameters and the mean values of the parameters estimated from 1000 replicates are given in **Tables A1 - A12**. The range of parameters from the 1000 replicates is shown in **Figures A1 - A12**. In these figures, a box represents the 25^{th} and 75^{th} percentiles, a horizontal line represents the median and the whiskers extend to the 2.5^{th} and 97.5^{th} percentiles of the generated parameters. The historical value is shown as a solid diamond.

Table A1 Comparison of historical and generated mean annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	695	710	694
Lissadell	616	619	
Mardie	276		285
Meedo	216	218	217
Perth	868	870	
Cuttening	312	314	
Norseman	287	289	287
Katherine	974	977	
Alice Springs	280	283	282
Marree	164	165	167
Orroroo	341	343	344
Wallaroo	360	361	361
Adelaide	530	530	
Eudunda	446	450	448
Palmerville	1034	1036	1041
Kalamia	1085	1098	1090
Emerald	642	646	646
Barcaldine	496	499	499
Cape Capricorn	801	808	803
Rockhampton	946	969	946
Cape Moreton	1550	1555	1553
Brisbane	1154	1161	1158
Pittsworth	703	705	
Miles	661	662	662
Cunnamulla	374	378	375
Wentworth	288	289	290
Balranald	322	324	325
Bingara	745	748	743
Mudgee	670	674	671
Sydney	1226	1230	1224
Moruya	972	984	972
Adelong	795	798	
Tumut	822	825	
Hay	369	372	371
Narraport	354	356	
Tongala	443	450	445
Caniambo	524	528	525
Orbost	855	857	854
Melbourne	657	657	
Toorourrong	804	806	
Meredith	685	686	
Frankford	1069	1074	1070
Fingal	611	618	611
Sandford	578	580	577



(a) AR(1)

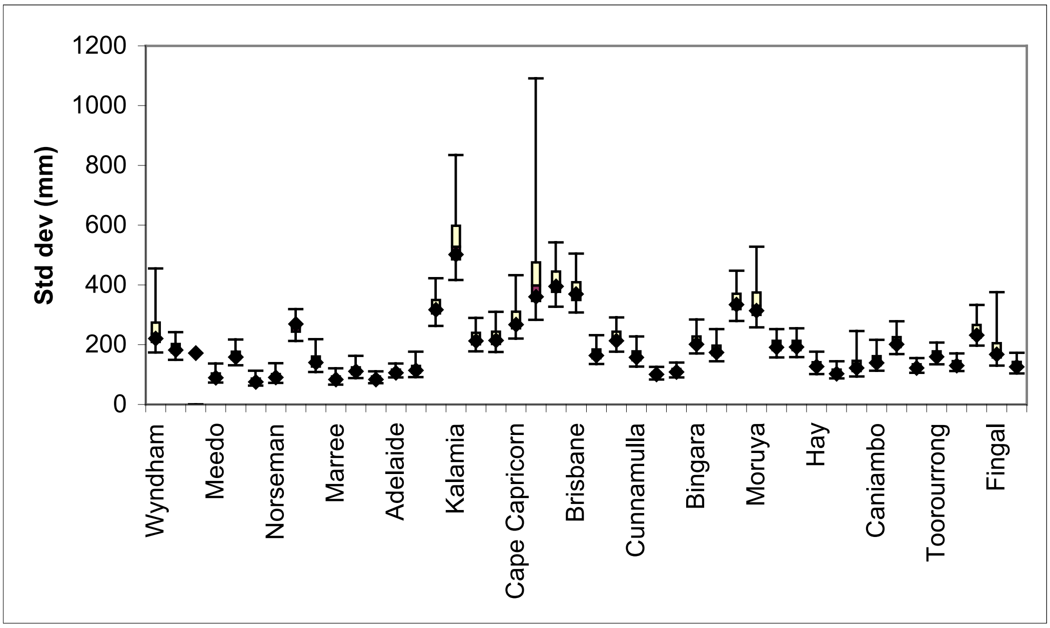


(b) HSM

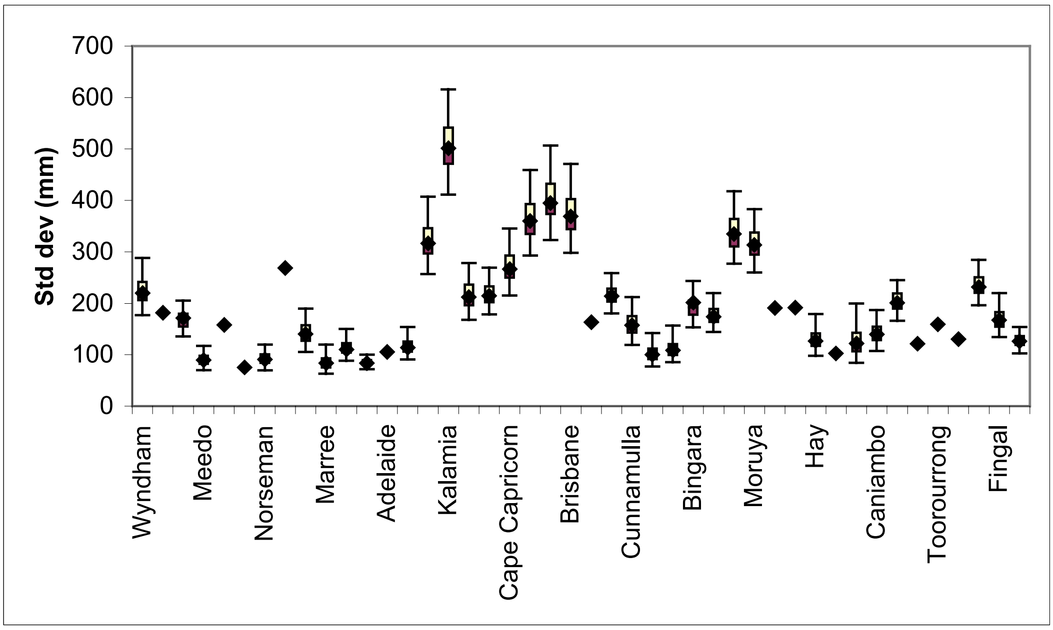
Figure A1 Comparison of mean annual rainfall.

Table A2 Comparison of historical and generated standard deviation of annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	220	270	225
Lissadell	182	189	
Mardie	172		168
Meedo	90	96	91
Perth	158	166	
Cuttening	76	81	
Norseman	91	95	93
Katherine	269	261	
Alice Springs	141	148	143
Marree	84	87	85
Orroroo	110	115	114
Wallaroo	83	87	85
Adelaide	106	109	
Eudunda	114	121	117
Palmerville	316	330	324
Kalamia	502	556	507
Emerald	212	222	218
Barcaldine	214	226	218
Cape Capricorn	267	292	273
Rockhampton	360	475	366
Cape Moreton	395	415	405
Brisbane	369	384	376
Pittsworth	163	172	
Miles	214	226	217
Cunnamulla	158	166	160
Wentworth	100	101	103
Balranald	109	112	113
Bingara	201	213	193
Mudgee	174	185	177
Sydney	335	348	339
Moruya	313	347	317
Adelong	191	199	
Tumut	191	200	
Hay	127	131	131
Narraport	103	109	
Tongala	122	142	127
Caniambo	140	150	143
Orbost	201	212	205
Melbourne	122	127	
Toorourrong	159	166	
Meredith	130	136	
Frankford	232	253	236
Fingal	167	198	170
Sandford	126	133	128



(a) AR(1)

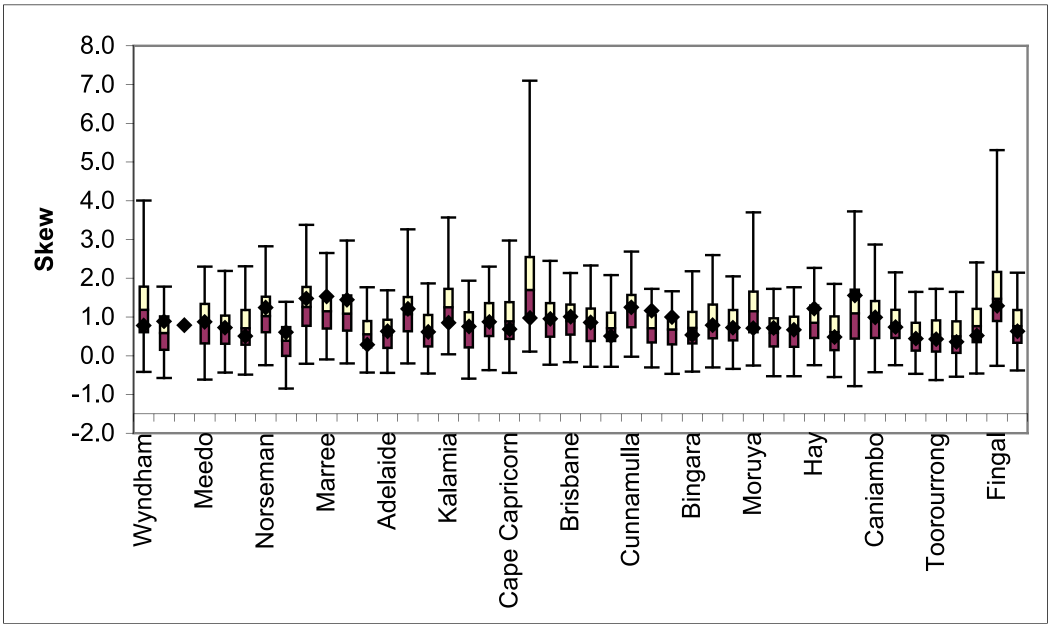


(b) HSM

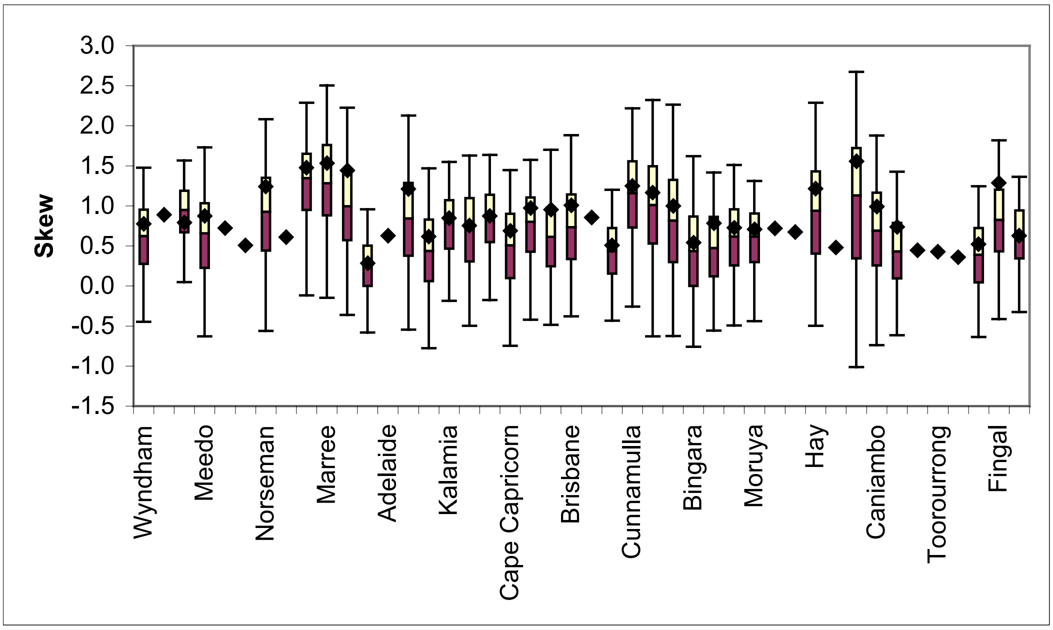
Figure A2 Comparison of the standard deviation of annual rainfall.

Table A3 Comparison of historical and generated skewness of annual rainfall.

Stations	Historical	AR(1)	mean
Wyndham	0.777	1.304	0.601
Lissadell	0.889	0.579	
Mardie	0.791		0.911
Meedo	0.876	0.824	0.614
Perth	0.724	0.716	
Cuttening	0.506	0.773	
Norseman	1.241	1.093	0.876
Katherine	0.610	0.191	
Alice Springs	1.478	1.326	1.273
Marree	1.533	1.180	1.286
Orroroo	1.443	1.156	0.988
Wallaroo	0.285	0.579	0.232
Adelaide	0.628	0.571	
Eudunda	1.211	1.168	0.830
Palmerville	0.617	0.659	0.431
Kalamia	0.847	1.353	0.762
Emerald	0.753	0.685	0.676
Barcaldine	0.876	0.954	0.827
Cape Capricorn	0.688	0.962	0.474
Rockhampton	0.972	2.094	0.744
Cape Moreton	0.954	0.979	0.606
Brisbane	1.009	0.938	0.742
Pittsworth	0.857	0.834	
Miles	0.506	0.770	0.425
Cunnamulla	1.249	1.202	1.116
Wentworth	1.167	0.711	0.985
Balranald	1.001	0.657	0.820
Bingara	0.541	0.757	0.415
Mudgee	0.785	0.937	0.469
Sydney	0.729	0.804	0.597
Moruya	0.708	1.238	0.570
Adelong	0.721	0.606	
Tumut	0.674	0.627	
Hay	1.217	0.886	0.916
Narraport	0.482	0.607	
Tongala	1.557	1.170	1.020
Caniambo	0.991	0.996	0.681
Orbost	0.738	0.869	0.426
Melbourne	0.447	0.515	
Toorourrong	0.427	0.529	
Meredith	0.359	0.505	
Frankford	0.522	0.837	0.370
Fingal	1.286	1.710	0.801
Sandford	0.630	0.786	0.623



(a) AR(1)

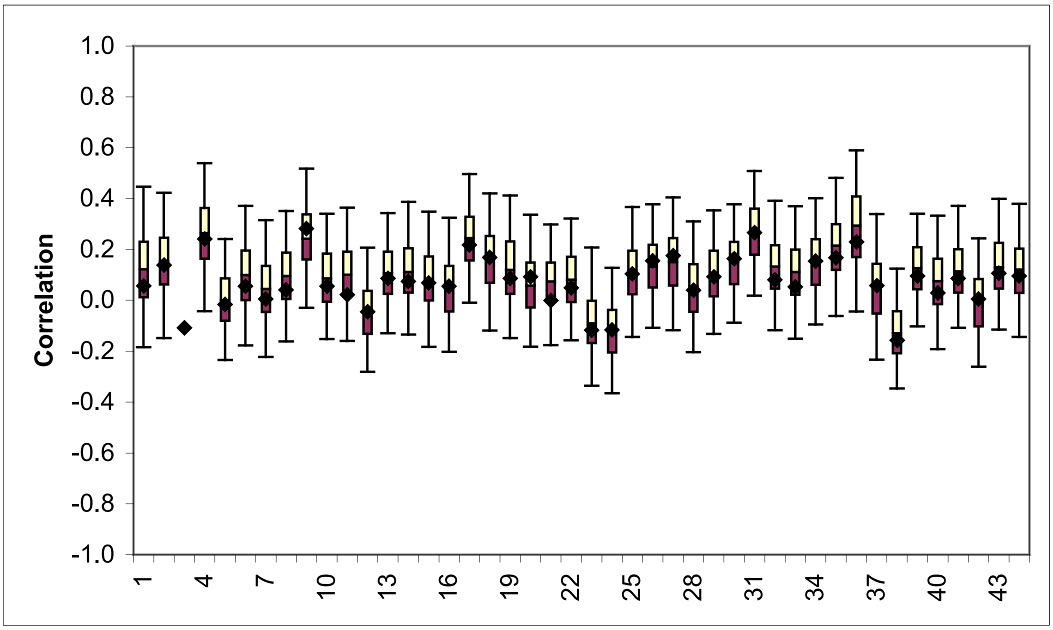


(b) HSM

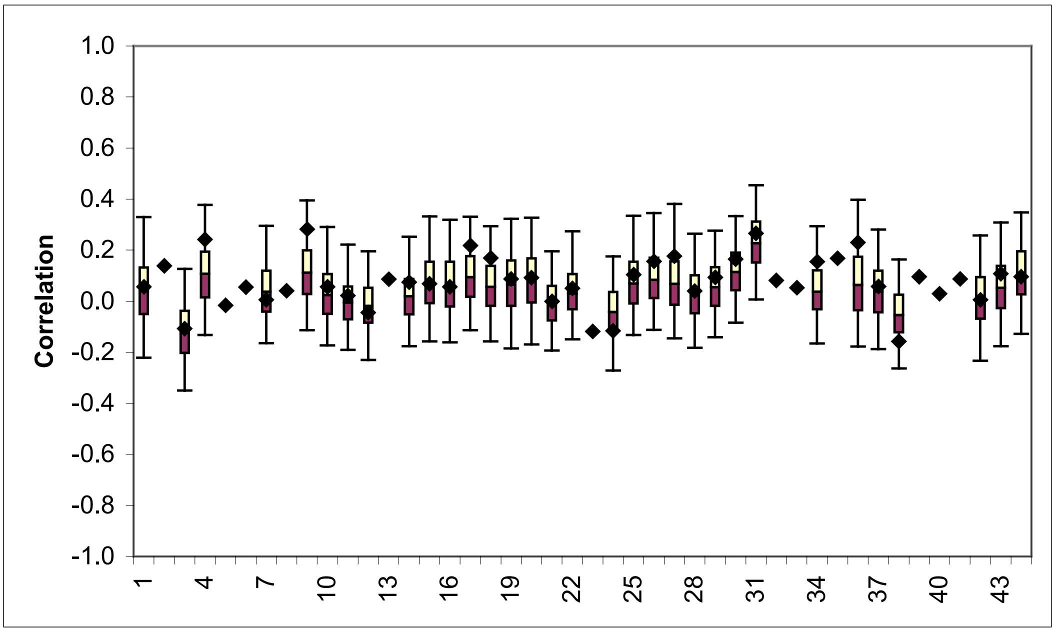
Figure A3 Comparison of the skewness of annual rainfall.

Table A4 Comparison of historical and generated lag one autocorrelation coefficient of annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	0.057	0.125	0.043
Lissadell	0.139	0.152	
Mardie	-0.108		-0.119
Meedo	0.242	0.263	0.107
Perth	-0.016	0.005	
Cuttening	0.055	0.100	
Norseman	0.005	0.046	0.045
Katherine	0.041	0.096	
Alice Springs	0.282	0.246	0.118
Marree	0.056	0.090	0.032
Orroroo	0.022	0.104	-0.001
Wallaroo	-0.045	-0.045	-0.018
Adelaide	0.086	0.110	
Eudunda	0.074	0.117	0.022
Palmerville	0.068	0.085	0.072
Kalamia	0.056	0.048	0.071
Emerald	0.218	0.243	0.099
Barcaldine	0.169	0.160	0.062
Cape Capricorn	0.087	0.128	0.071
Rockhampton	0.092	0.063	0.083
Cape Moreton	0.000	0.070	-0.006
Brisbane	0.050	0.081	0.039
Pittsworth	-0.118	-0.084	
Miles	-0.116	-0.120	-0.044
Cunnamulla	0.104	0.110	0.077
Wentworth	0.156	0.134	0.094
Balranald	0.176	0.150	0.078
Bingara	0.040	0.053	0.029
Mudgee	0.093	0.107	0.060
Sydney	0.164	0.146	0.119
Moruya	0.266	0.270	0.230
Adelong	0.081	0.133	
Tumut	0.053	0.111	
Hay	0.155	0.152	0.047
Narraport	0.168	0.209	
Tongala	0.230	0.288	0.074
Caniambo	0.058	0.050	0.040
Orbost	-0.157	-0.124	-0.050
Melbourne	0.096	0.123	
Toorourrong	0.029	0.075	
Meredith	0.087	0.118	
Frankford	0.005	-0.003	0.013
Fingal	0.107	0.137	0.057
Sandford	0.096	0.117	0.109



(a) AR(1)

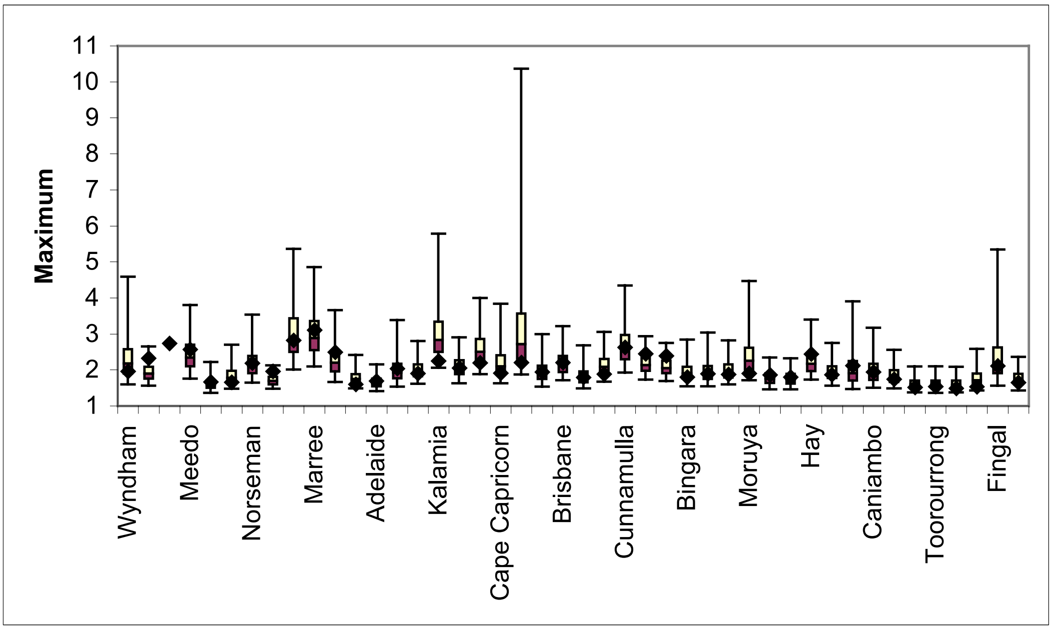


(b) HSM

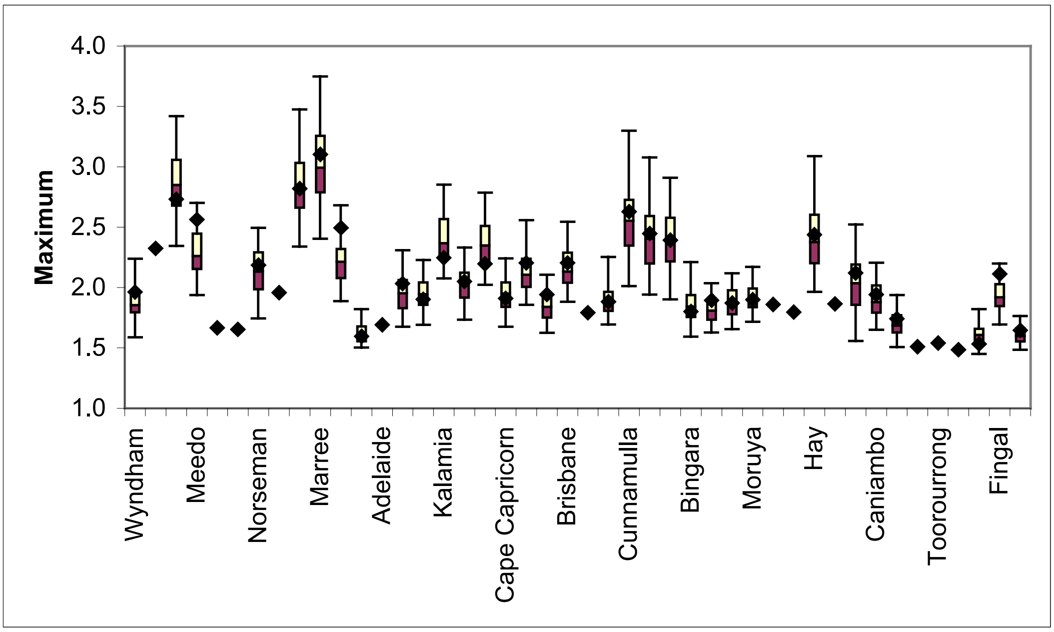
Figure A4 Comparison of the lag one autocorrelation of annual rainfall.

Table A5 Comparison of historical and generated annual maximum rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	1.963	2.584	1.872
Lissadell	2.324	1.946	
Mardie	2.732		2.876
Meedo	2.564	2.469	2.303
Perth	1.667	1.644	
Cuttening	1.654	1.859	
Norseman	2.185	2.233	2.133
Katherine	1.957	1.720	
Alice Springs	2.820	3.145	2.850
Marree	3.104	3.031	3.020
Orroroo	2.495	2.320	2.217
Wallaroo	1.596	1.792	1.621
Adelaide	1.691	1.675	
Eudunda	2.033	2.077	1.960
Palmerville	1.901	2.027	1.950
Kalamia	2.245	3.092	2.412
Emerald	2.051	2.119	2.021
Barcaldine	2.196	2.624	2.372
Cape Capricorn	1.909	2.270	1.942
Rockhampton	2.204	3.628	2.138
Cape Moreton	1.941	2.011	1.848
Brisbane	2.203	2.216	2.157
Pittsworth	1.792	1.852	
Miles	1.883	2.167	1.903
Cunnamulla	2.627	2.728	2.571
Wentworth	2.447	2.190	2.406
Balranald	2.392	2.112	2.400
Bingara	1.802	1.967	1.848
Mudgee	1.894	2.014	1.813
Sydney	1.873	2.016	1.875
Moruya	1.900	2.444	1.913
Adelong	1.860	1.796	
Tumut	1.797	1.781	
Hay	2.438	2.264	2.415
Narraport	1.864	1.978	
Tongala	2.120	2.196	2.038
Caniambo	1.941	2.020	1.906
Orbost	1.740	1.882	1.706
Melbourne	1.511	1.631	
Toorourrong	1.541	1.633	
Meredith	1.486	1.627	
Frankford	1.533	1.844	1.613
Fingal	2.112	2.576	1.934
Sandford	1.646	1.793	1.611



(a) AR(1)

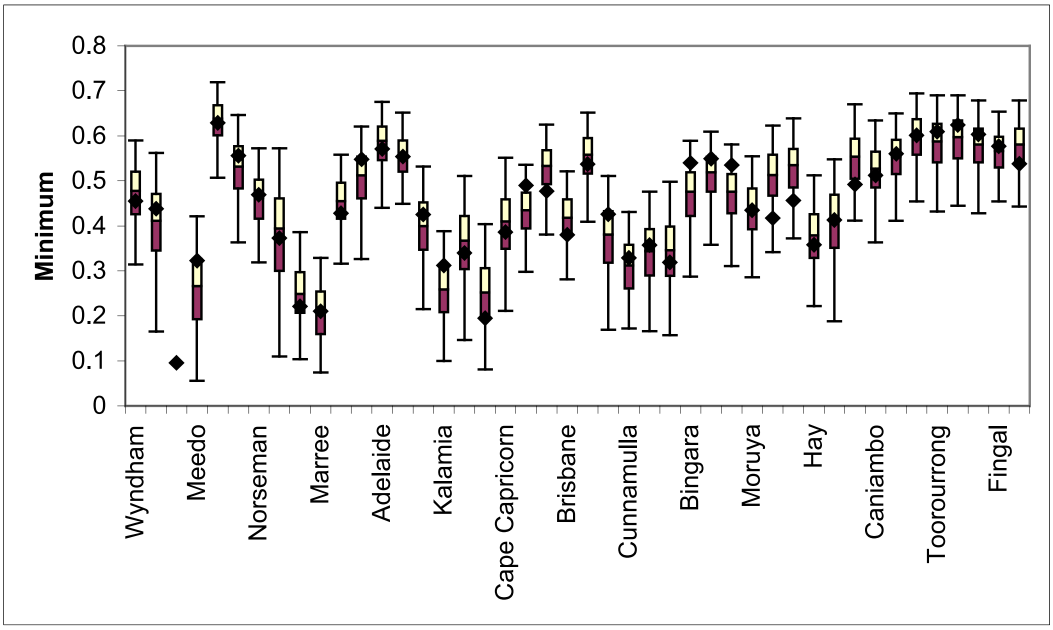


(b) HSM

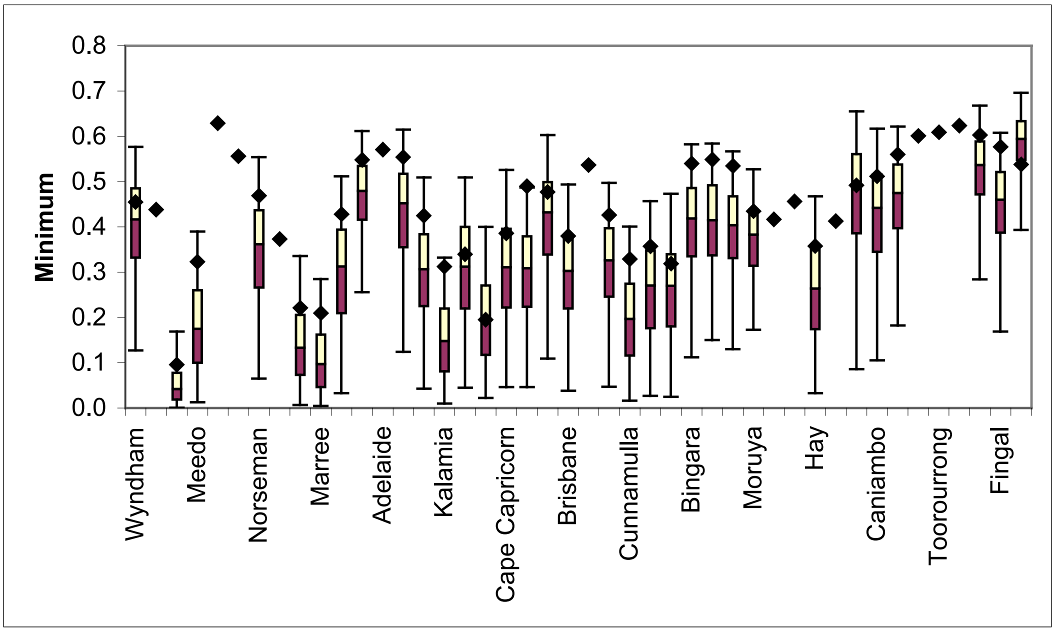
Figure A5 Comparison of the maximum annual rainfall.

Table A6 Comparison of minimum annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	0.455	0.472	0.400
Lissadell	0.438	0.402	
Mardie	0.096		0.056
Meedo	0.323	0.257	0.183
Perth	0.629	0.630	
Cuttening	0.556	0.525	
Norseman	0.469	0.458	0.343
Katherine	0.373	0.377	
Alice Springs	0.221	0.250	0.144
Marree	0.210	0.207	0.111
Orroroo	0.428	0.453	0.298
Wallaroo	0.548	0.502	0.469
Adelaide	0.571	0.580	
Eudunda	0.554	0.554	0.427
Palmerville	0.425	0.395	0.300
Kalamia	0.312	0.257	0.155
Emerald	0.340	0.359	0.305
Barcaldine	0.195	0.249	0.197
Cape Capricorn	0.386	0.401	0.305
Rockhampton	0.490	0.431	0.295
Cape Moreton	0.477	0.527	0.409
Brisbane	0.380	0.414	0.293
Pittsworth	0.537	0.552	
Miles	0.426	0.372	0.312
Cunnamulla	0.329	0.308	0.198
Wentworth	0.357	0.339	0.259
Balranald	0.319	0.340	0.260
Bingara	0.540	0.465	0.401
Mudgee	0.549	0.510	0.404
Sydney	0.535	0.468	0.391
Moruya	0.435	0.435	0.371
Adelong	0.417	0.507	
Tumut	0.456	0.525	
Hay	0.358	0.376	0.261
Narraport	0.413	0.404	
Tongala	0.492	0.549	0.456
Caniambo	0.512	0.519	0.421
Orbost	0.560	0.548	0.458
Melbourne	0.601	0.593	
Toorourrong	0.609	0.581	
Meredith	0.624	0.588	
Frankford	0.603	0.574	0.522
Fingal	0.577	0.563	0.443
Sandford	0.538	0.574	0.581



(a) AR(1)

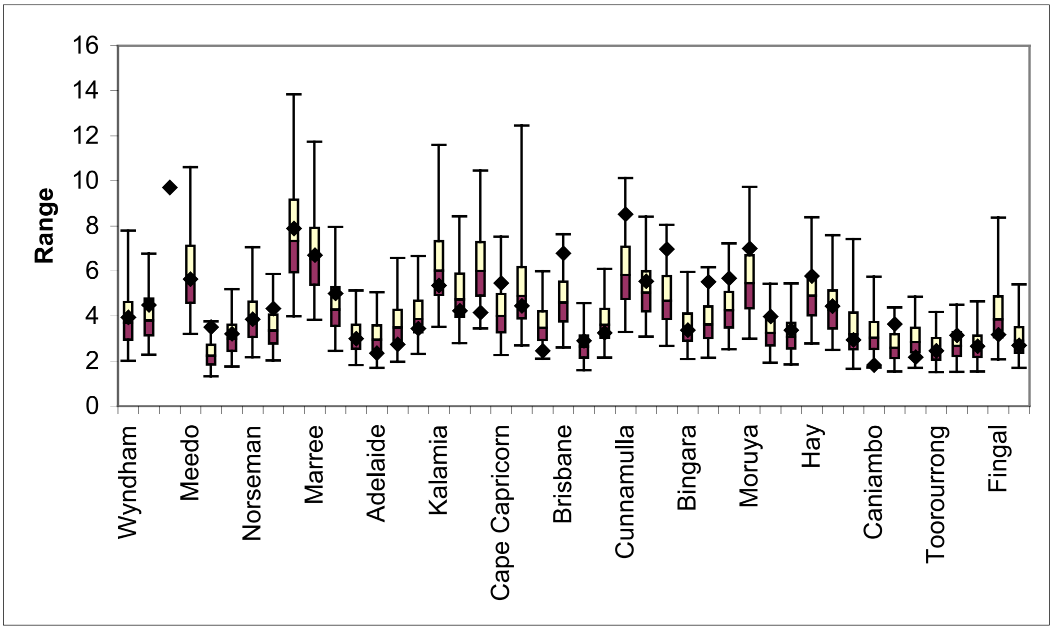


(b) HSM

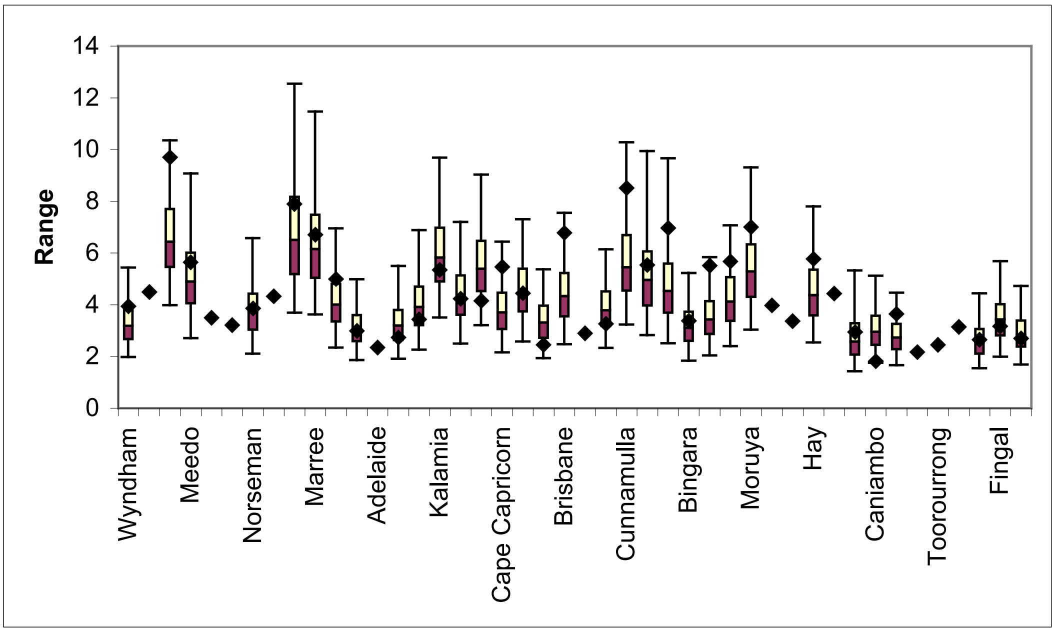
Figure A6. Comparison of the minimum annual rainfall.

Table A7 Comparison of the range of annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	3.935	4.176	3.338
Lissadell	4.494	4.043	
Mardie	9.699		6.657
Meedo	5.635	6.043	5.139
Perth	3.499	2.332	
Cuttening	3.211	3.108	
Norseman	3.855	3.940	3.847
Katherine	4.325	3.493	
Alice Springs	7.892	7.764	6.922
Marree	6.704	6.839	6.521
Orroroo	4.999	4.552	4.200
Wallaroo	2.987	3.123	3.147
Adelaide	2.341	3.079	
Eudunda	2.730	3.704	3.316
Palmerville	3.439	4.035	4.085
Kalamia	5.348	6.351	6.049
Emerald	4.233	5.015	4.444
Barcaldine	4.147	6.260	5.613
Cape Capricorn	5.461	4.250	3.838
Rockhampton	4.443	5.647	4.610
Cape Moreton	2.453	3.661	3.394
Brisbane	6.783	4.730	4.515
Pittsworth	2.897	2.727	
Miles	3.255	3.752	3.911
Cunnamulla	8.518	6.042	5.799
Wentworth	5.534	5.218	5.217
Balranald	6.970	4.904	4.858
Bingara	3.374	3.598	3.232
Mudgee	5.516	3.788	3.584
Sydney	5.674	4.403	4.294
Moruya	7.000	5.680	5.489
Adelong	3.972	3.408	
Tumut	3.370	3.215	
Hay	5.774	5.077	4.599
Narraport	4.433	4.428	
Tongala	2.943	3.644	2.785
Caniambo	1.817	3.226	3.095
Orbost	3.645	2.717	2.821
Melbourne	2.172	2.979	
Toorourrong	2.450	2.589	
Meredith	3.137	2.782	
Frankford	2.652	2.815	2.653
Fingal	3.164	4.266	3.520
Sandford	2.694	3.051	2.931



(a) AR(1)

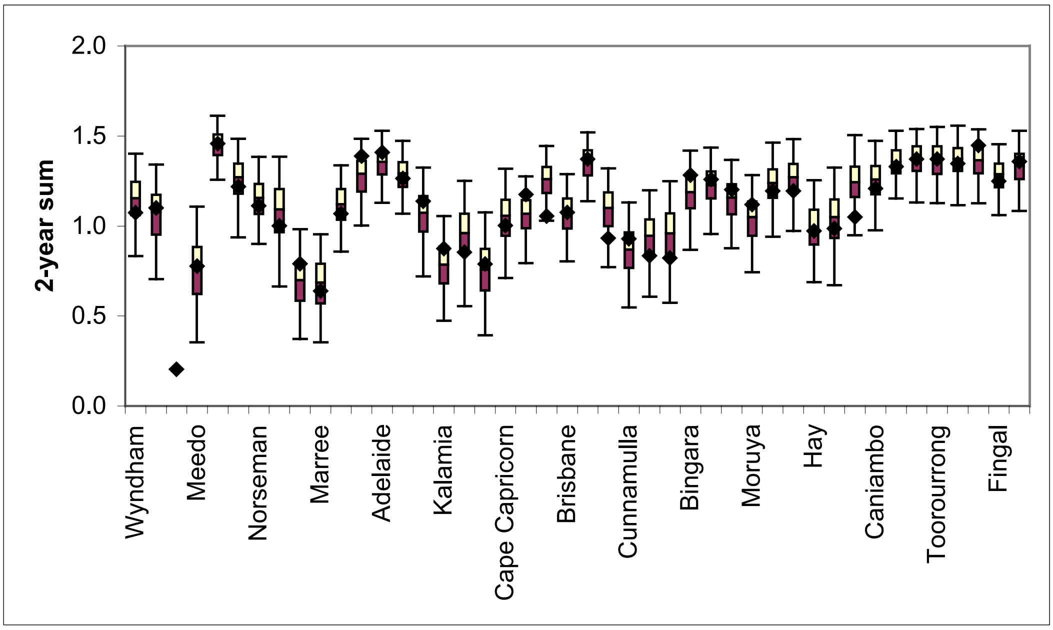


(b) HSM

Figure A7 Comparison of the range of annual rainfall.

Table A8 Comparison of the minimum 2-year annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	1.073	1.145	1.112
Lissadell	1.100	1.059	
Mardie	0.204		0.469
Meedo	0.777	0.752	0.783
Perth	1.458	1.448	
Cuttening	1.218	1.257	
Norseman	1.112	1.148	1.077
Katherine	1.001	1.077	
Alice Springs	0.791	0.695	0.699
Marree	0.638	0.676	0.643
Orroroo	1.069	1.115	1.018
Wallaroo	1.389	1.273	1.230
Adelaide	1.408	1.348	
Eudunda	1.263	1.282	1.201
Palmerville	1.138	1.060	1.006
Kalamia	0.874	0.783	0.713
Emerald	0.854	0.950	0.999
Barcaldine	0.788	0.753	0.793
Cape Capricorn	1.003	1.039	0.994
Rockhampton	1.174	1.061	0.947
Cape Moreton	1.054	1.253	1.172
Brisbane	1.075	1.068	0.991
Pittsworth	1.371	1.348	
Miles	0.932	1.087	1.007
Cunnamulla	0.929	0.861	0.823
Wentworth	0.834	0.936	0.941
Balranald	0.822	0.947	0.949
Bingara	1.283	1.175	1.156
Mudgee	1.258	1.222	1.154
Sydney	1.201	1.145	1.129
Moruya	1.119	1.041	1.045
Adelong	1.194	1.228	
Tumut	1.195	1.259	
Hay	0.972	0.987	0.948
Narraport	0.985	1.038	
Tongala	1.049	1.242	1.238
Caniambo	1.208	1.252	1.187
Orbost	1.330	1.350	1.236
Melbourne	1.372	1.367	
Toorourrong	1.372	1.359	
Meredith	1.347	1.363	
Frankford	1.447	1.355	1.316
Fingal	1.248	1.278	1.201
Sandford	1.358	1.327	1.377



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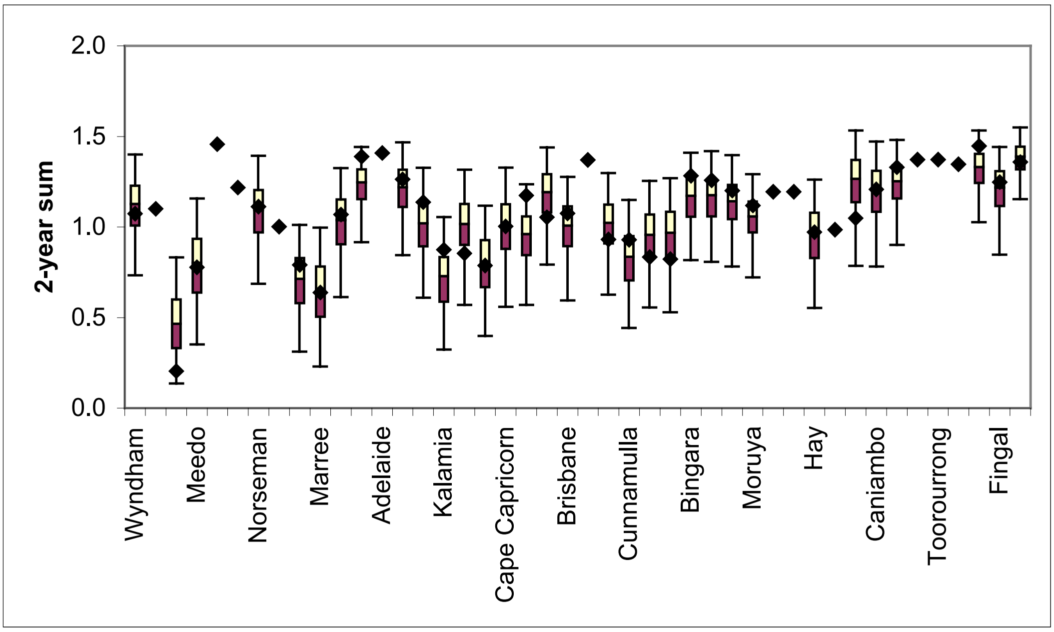
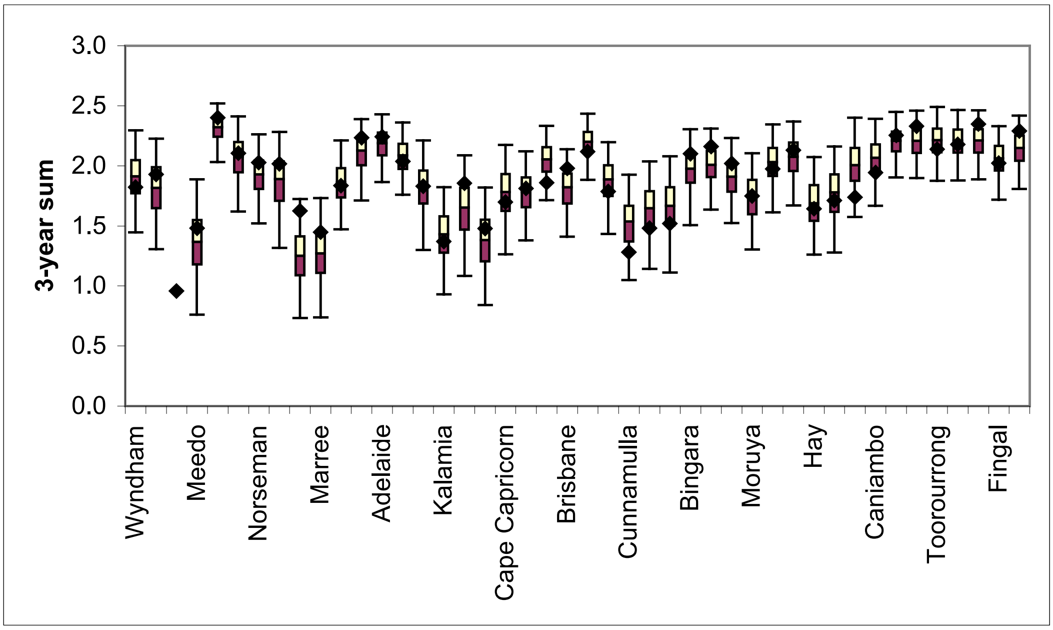


(b) HSM

Figure A8 Comparison of the minimum 2-year annual rainfall.

TableA9 Comparison of the minimum 3-year annual rainfall.

Station	Historical	AR(1)	HSM
Wyndham	1.823	1.902	1.897
Lissadell	1.929	1.809	
Mardie	0.957		1.086
Meedo	1.480	1.358	1.497
Perth	2.402	2.314	
Cuttening	2.105	2.065	
Norseman	2.028	1.918	1.869
Katherine	2.016	1.862	
Alice Springs	1.626	1.248	1.370
Marree	1.449	1.272	1.324
Orroroo	1.836	1.860	1.806
Wallaroo	2.235	2.104	2.057
Adelaide	2.242	2.176	
Eudunda	2.036	2.078	2.033
Palmerville	1.831	1.812	1.793
Kalamia	1.369	1.419	1.373
Emerald	1.854	1.635	1.768
Barcaldine	1.479	1.375	1.486
Cape Capricorn	1.698	1.767	1.763
Rockhampton	1.813	1.775	1.676
Cape Moreton	1.861	2.049	1.993
Brisbane	1.979	1.805	1.763
Pittsworth	2.118	2.191	
Miles	1.788	1.871	1.780
Cunnamulla	1.282	1.519	1.540
Wentworth	1.480	1.638	1.708
Balranald	1.518	1.650	1.722
Bingara	2.100	1.963	1.976
Mudgee	2.162	2.005	1.969
Sydney	2.019	1.902	1.921
Moruya	1.749	1.733	1.775
Adelong	1.974	2.023	
Tumut	2.131	2.066	
Hay	1.644	1.687	1.721
Narraport	1.712	1.765	
Tongala	1.738	2.007	2.082
Caniambo	1.943	2.058	2.015
Orbost	2.255	2.200	2.075
Melbourne	2.330	2.201	
Toorourrong	2.139	2.207	
Meredith	2.178	2.200	
Frankford	2.347	2.199	2.160
Fingal	2.021	2.057	2.013
Sandford	2.291	2.142	2.214



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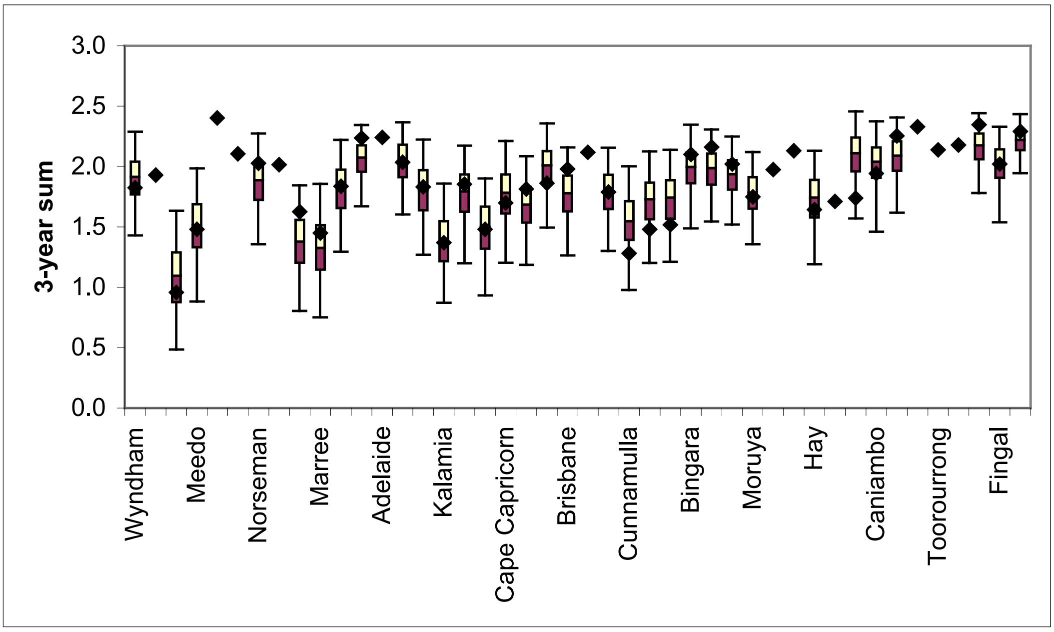
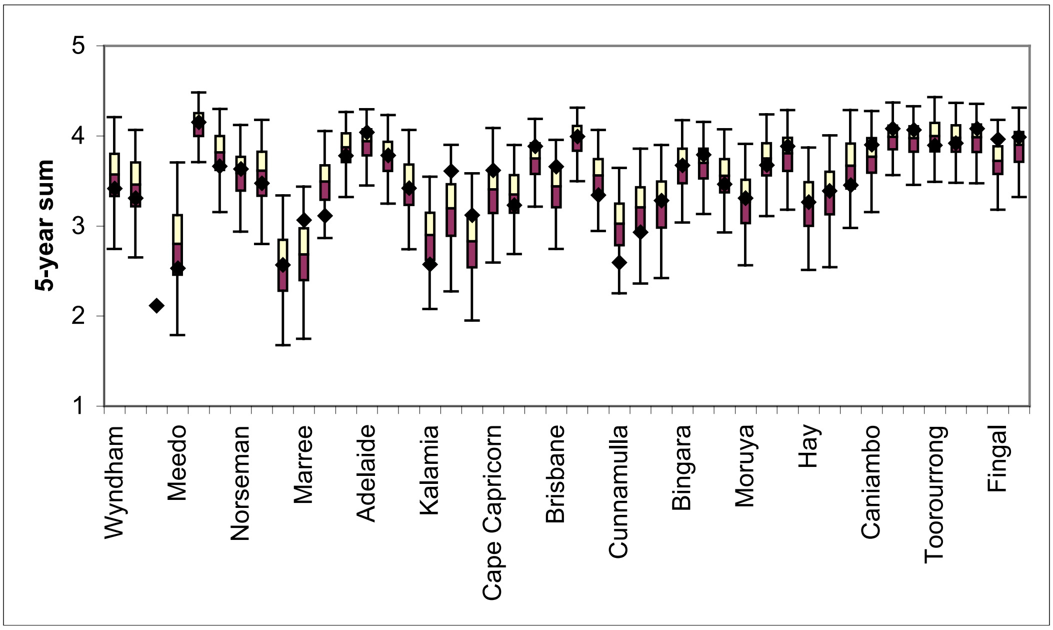


(b) HSM

Figure A9 Comparison of the minimum 3-year annual rainfall.

Table A10 Comparison of the minimum 5-year annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	3.416	3.552	3.568
Lissadell	3.307	3.441	
Mardie	2.115		2.547
Meedo	2.529	2.789	3.058
Perth	4.151	4.121	
Cuttening	3.661	3.789	
Norseman	3.633	3.572	3.556
Katherine	3.473	3.562	
Alice Springs	2.568	2.556	2.842
Marree	3.065	2.665	2.826
Orroroo	3.113	3.477	3.495
Wallaroo	3.779	3.859	3.804
Adelaide	4.041	3.922	
Eudunda	3.784	3.773	3.780
Palmerville	3.418	3.449	3.469
Kalamia	2.574	2.871	2.856
Emerald	3.608	3.171	3.420
Barcaldine	3.121	2.810	3.023
Cape Capricorn	3.618	3.383	3.407
Rockhampton	3.232	3.343	3.265
Cape Moreton	3.883	3.742	3.723
Brisbane	3.658	3.410	3.433
Pittsworth	3.993	3.962	
Miles	3.341	3.547	3.454
Cunnamulla	2.595	3.011	3.107
Wentworth	2.932	3.184	3.348
Balranald	3.283	3.222	3.385
Bingara	3.671	3.657	3.710
Mudgee	3.790	3.683	3.683
Sydney	3.462	3.545	3.595
Moruya	3.310	3.265	3.334
Adelong	3.676	3.733	
Tumut	3.885	3.788	
Hay	3.266	3.239	3.386
Narraport	3.390	3.364	
Tongala	3.453	3.664	3.840
Caniambo	3.901	3.768	3.762
Orbost	4.081	3.979	3.845
Melbourne	4.067	3.957	
Toorourrong	3.896	3.982	
Meredith	3.918	3.963	
Frankford	4.079	3.969	3.939
Fingal	3.961	3.717	3.721
Sandford	3.987	3.875	3.961



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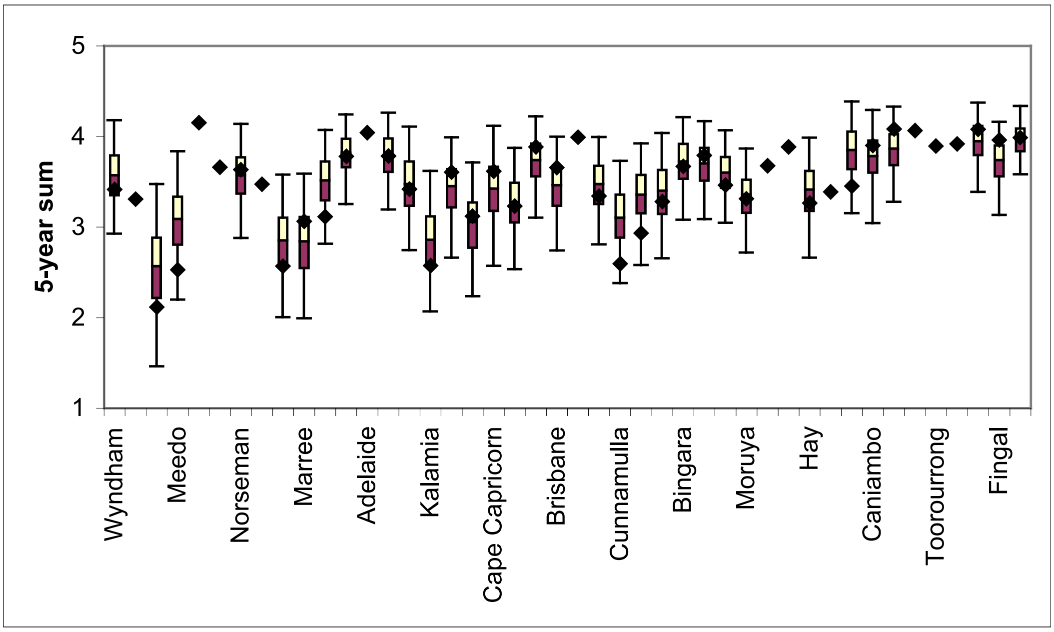
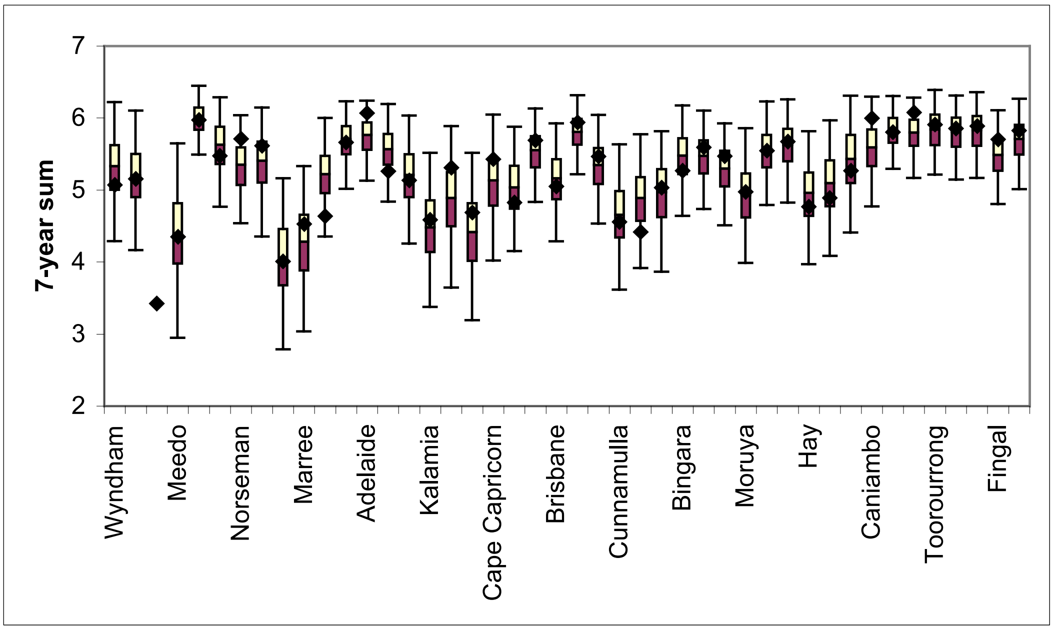


(b) HSM

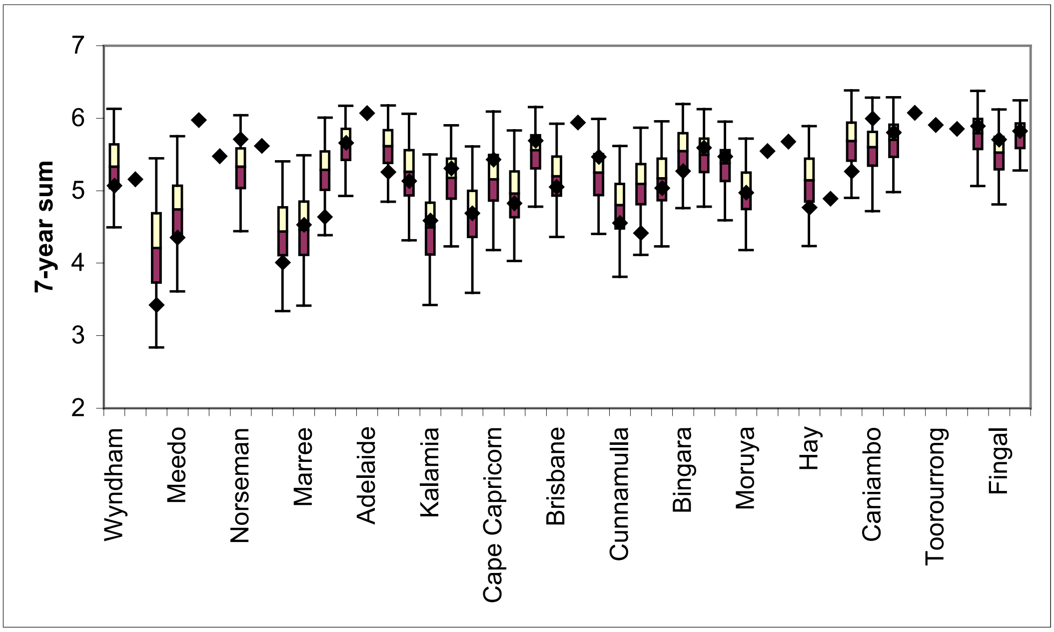
Figure A10 Comparison of the minimum 5-year annual rainfall.

Table A11 Comparison of the minimum 7-year annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	5.069	5.310	5.333
Lissadell	5.157	5.184	
Mardie	3.423		4.191
Meedo	4.352	4.389	4.724
Perth	5.974	5.985	
Cuttening	5.477	5.606	
Norseman	5.709	5.329	5.304
Katherine	5.616	5.362	
Alice Springs	4.008	4.050	4.427
Marree	4.529	4.251	4.467
Orroroo	4.639	5.203	5.263
Wallaroo	5.662	5.682	5.620
Adelaide	6.070	5.742	
Eudunda	5.259	5.556	5.597
Palmerville	5.132	5.190	5.242
Kalamia	4.587	4.484	4.478
Emerald	5.310	4.852	5.153
Barcaldine	4.687	4.411	4.672
Cape Capricorn	5.429	5.105	5.164
Rockhampton	4.827	5.034	4.948
Cape Moreton	5.688	5.531	5.531
Brisbane	5.051	5.138	5.186
Pittsworth	5.940	5.802	
Miles	5.467	5.332	5.227
Cunnamulla	4.555	4.650	4.780
Wentworth	4.418	4.871	5.080
Balranald	5.034	4.936	5.150
Bingara	5.269	5.457	5.521
Mudgee	5.595	5.459	5.478
Sydney	5.472	5.284	5.338
Moruya	4.972	4.921	4.991
Adelong	5.546	5.528	
Tumut	5.675	5.608	
Hay	4.769	4.935	5.130
Narraport	4.889	5.084	
Tongala	5.268	5.424	5.670
Caniambo	5.996	5.580	5.570
Orbost	5.802	5.823	5.681
Melbourne	6.076	5.784	
Toorourrong	5.908	5.829	
Meredith	5.853	5.793	
Frankford	5.890	5.814	5.777
Fingal	5.704	5.482	5.511
Sandford	5.824	5.691	5.761



(a) AR(1)

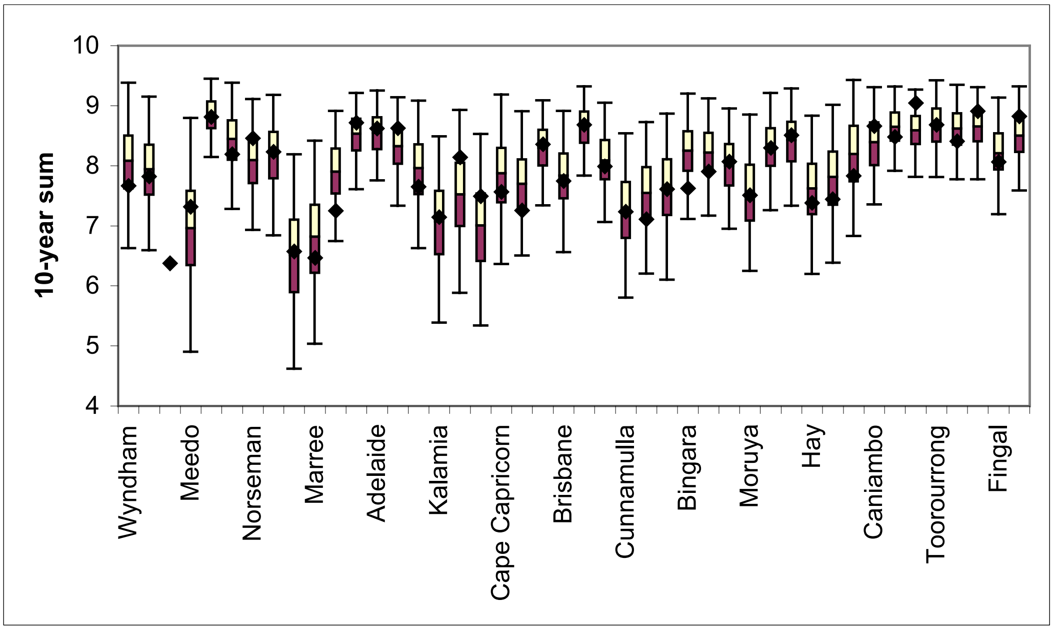


(b) HSM

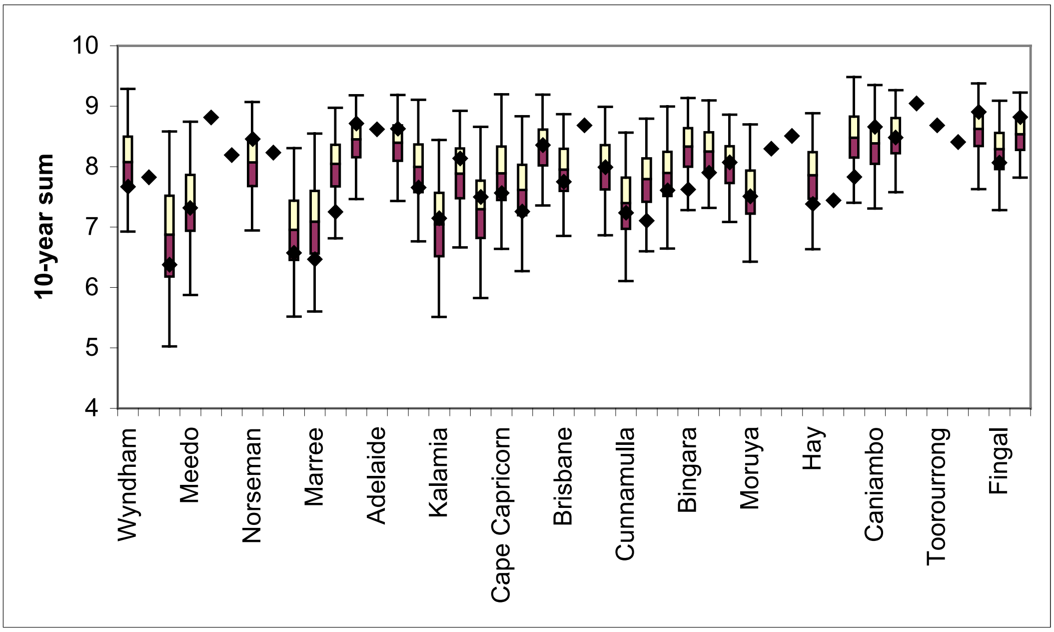
Figure A11 Comparison of the minimum 7-year annual rainfall.

Table A12 Comparison of the minimum 10-year annual rainfall.

Stations	Historical	AR(1)	HSM
Wyndham	7.667	8.061	8.075
Lissadell	7.822	7.915	
Mardie	6.377		6.829
Meedo	7.318	6.947	7.375
Perth	8.813	8.836	
Cuttening	8.192	8.415	
Norseman	8.458	8.072	8.047
Katherine	8.23	8.156	
Alice Springs	6.571	6.483	6.943
Marree	6.469	6.775	7.083
Orroroo	7.250	7.899	7.999
Wallaroo	8.715	8.496	8.418
Adelaide	8.620	8.536	
Eudunda	8.627	8.316	8.383
Palmerville	7.651	7.923	7.974
Kalamia	7.144	7.058	7.037
Emerald	8.138	7.499	7.866
Barcaldine	7.495	6.974	7.283
Cape Capricorn	7.564	7.823	7.897
Rockhampton	7.257	7.701	7.599
Cape Moreton	8.356	8.289	8.312
Brisbane	7.748	7.823	7.924
Pittsworth	8.682	8.629	
Miles	7.988	8.089	7.967
Cunnamulla	7.234	7.263	7.387
Wentworth	7.107	7.530	7.761
Balranald	7.610	7.621	7.872
Bingara	7.623	8.225	8.298
Mudgee	7.902	8.217	8.240
Sydney	8.071	8.000	8.034
Moruya	7.510	7.550	7.569
Adelong	8.296	8.300	
Tumut	8.507	8.398	
Hay	7.382	7.593	7.840
Narraport	7.440	7.781	
Tongala	7.831	8.191	8.476
Caniambo	8.659	8.370	8.359
Orbost	8.480	8.654	8.491
Melbourne	9.044	8.583	
Toorourrong	8.683	8.666	
Meredith	8.407	8.607	
Frankford	8.906	8.642	8.597
Fingal	8.065	8.218	8.254
Sandford	8.821	8.496	8.529

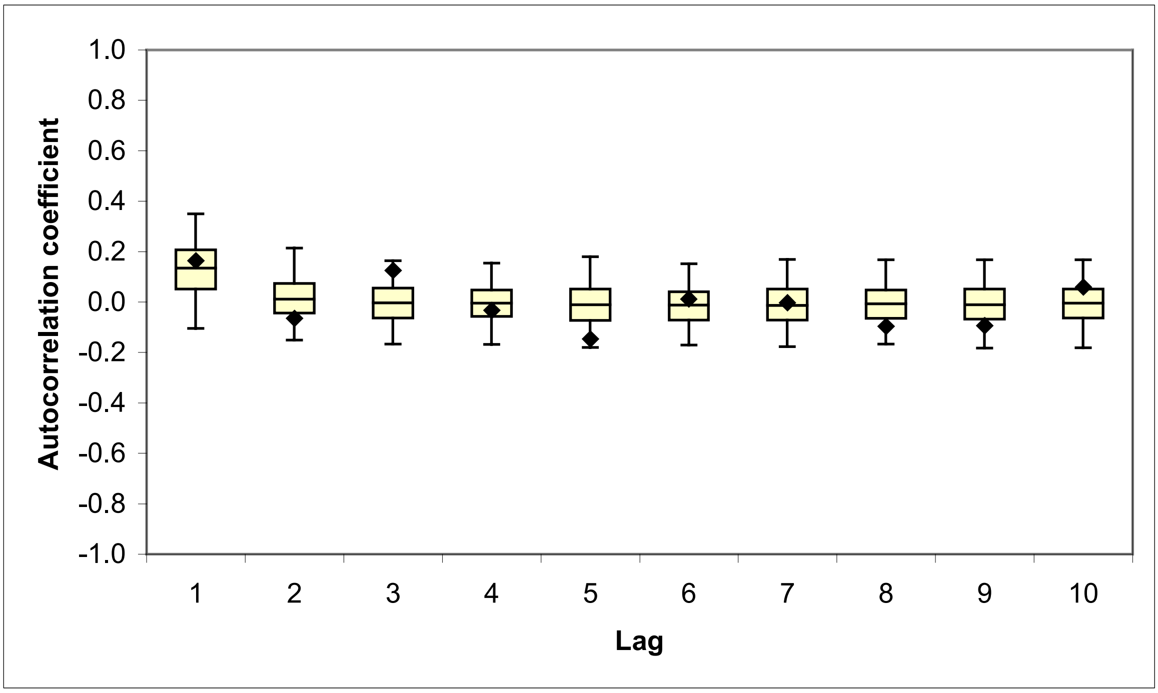


(a) AR(1)

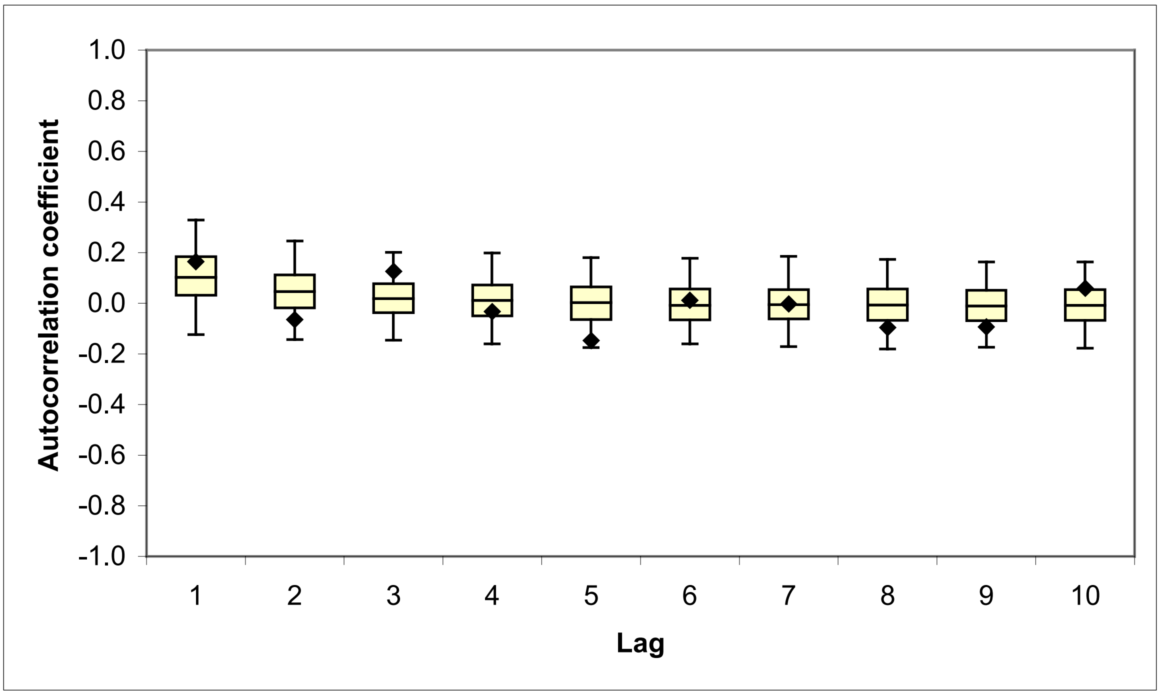


(b) HSM

Figure A12 Comparison of the minimum 10-year annual rainfall.

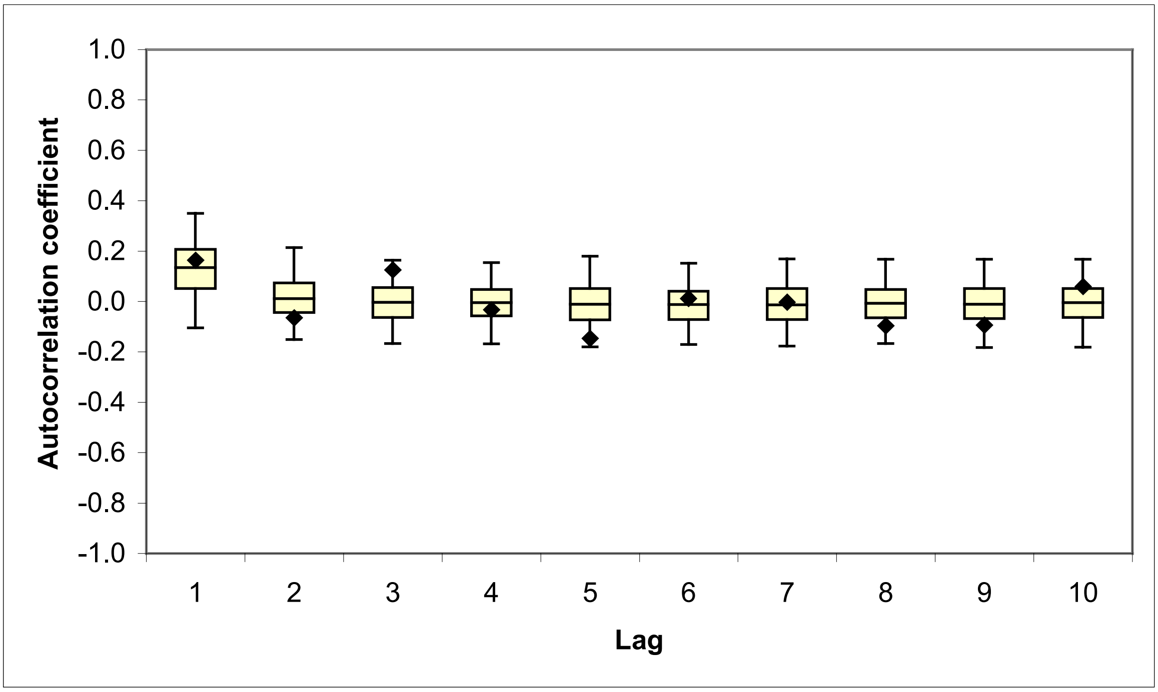


(a) AR(1)



(b) HSM

Figure A13 Autocorrelation function of annual rainfall for Wyndham.



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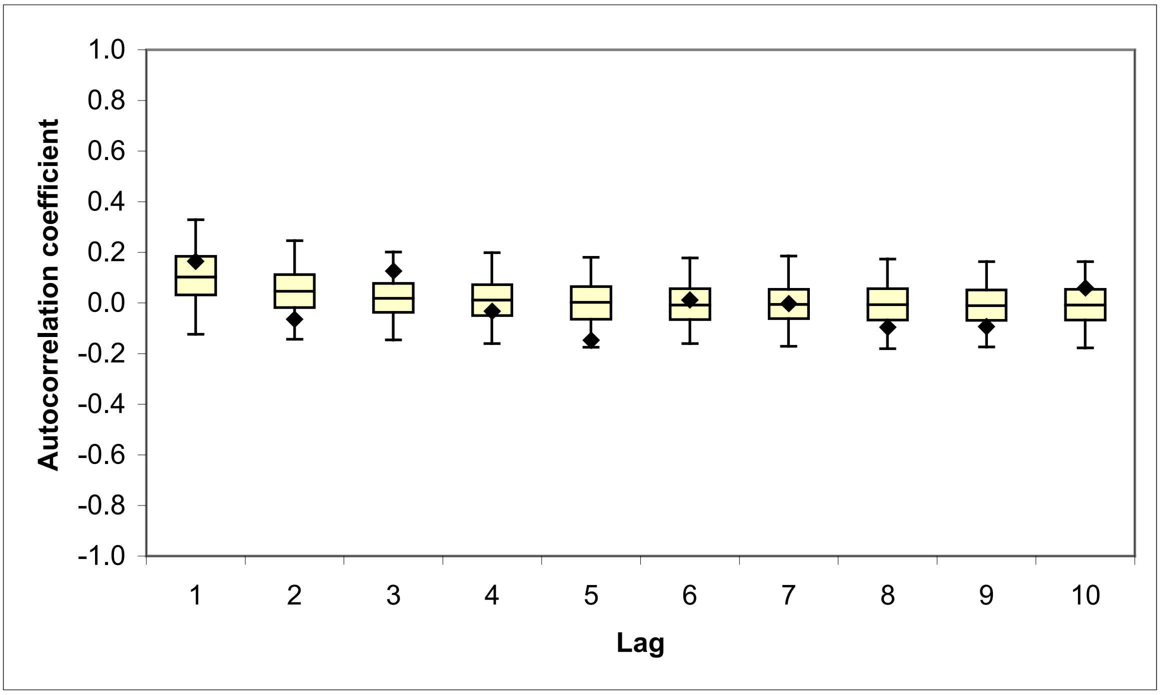

(b) HSM

Figure A14 Autocorrelation function of annual rainfall for Sydney.

CENTRE OFFICE

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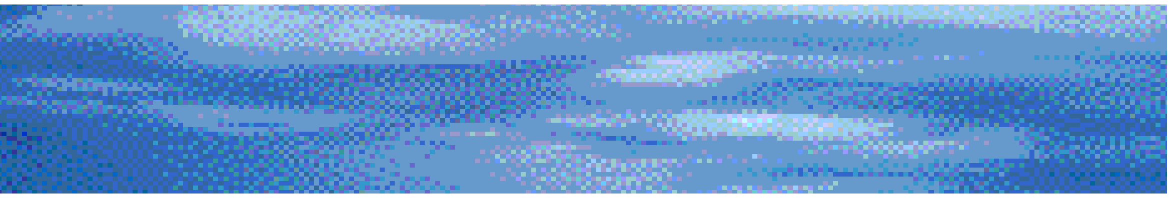

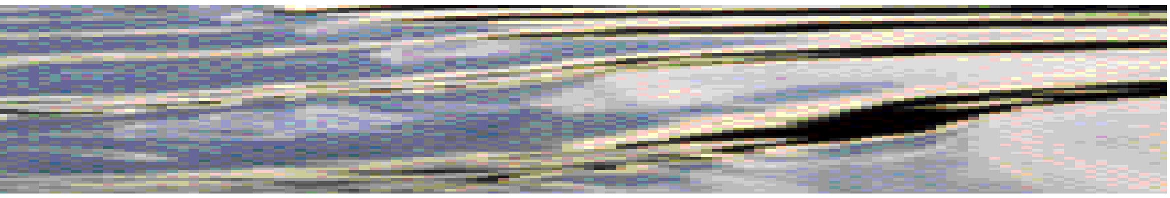

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- Brisbane City Council
- Bureau of Meteorology
- CSIRO Land and Water
- Department of Land and Water Conservation, NSW
- Department of Natural Resources and Environment, Vic
- Goulburn-Murray Water
- Griffith University
- Melbourne Water
- Monash University
- Murray-Darling Basin Commission
- Natural Resources and Mines, Qld
- Southern Rural Water
- The University of Melbourne
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- SA Water
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